

## SECOND SEMESTER MSc MATHEMATICS

### ADVANCED TOPOLOGY

#### OBJECTIVE QUESTIONS

1. If every open cover of a topological space has a finite subcover then the space is  
(a) Lindeloff (b) Normal (c) Compact (d) Complete
2. Every map from a compact space into a  $T_2$  space is  
(a) Closed (b) Constant (c) Identity (d) Open
3. A continuous bijection from a compact space onto a Hausdorff space is  
(a) Isomorphism (b) Homomorphism (c) Homeomorphism (d) Constant
4. Every continuous one-to-one from a compact space into a Hausdorff space is  
(a) Embedding (b) Connected (c) Isomorphism (d) Homomorphism
5. Which of the following is not true for a compact Hausdorff space  
(a) Normal (b)  $T_3$  (c) Regular (d) Completely regular
6. If a space is regular and Lindeloff then it is  
(a) Normal (b) Compact (c) Connected (d) Closed
7. If every open cover of a topological space has a countable subcover then the space is  
(a) Lindeloff (b) Normal (c) Compact (d) Complete
8. In a discrete space, every continuous real valued function is  
(a) Closed (b) Constant (c) Identity (d) Open
9. A subset of  $\mathbb{R}$  with usual topology is closed and bounded then the subset is  
(a) Regular (b) Normal (c) Complete (d) Compact
10. If a space  $X$  is regular and second countable then  $X$  is  
(a) Normal (b) Compact (c) Completely regular (d) Tychonoff
11. A  $T_4$  Space is  
(a) Normal and  $T_1$  (b) Normal and  $T_0$  (c) Regular and  $T_1$  (d) Regular and  $T_0$
12. A  $T_3$  space is  
(a) Normal and  $T_1$  (b) Normal and  $T_0$  (c) Regular and  $T_1$  (d) Regular and  $T_0$
13. If a space  $X$  has the property that for every two mutually disjoint closed subsets  $A$  and  $B$  of  $X$ , there exist a continuous function  $f: X \rightarrow [0, 1]$  such that  $f(A) = \{0\}$  and  $f(B) = \{1\}$  then  $X$  is  
(a) Normal (b) Regular (c) Lindeloff (d) Metrizable

14. Which of the following is related to the statement given below.

If a space  $X$  has the property that for every two mutually disjoint closed subsets  $A$  and  $B$  of  $X$ , there exist a continuous function  $f: X \rightarrow [0, 1]$  such that  $f(A) = \{0\}$  and  $f(B) = \{1\}$  then  $X$  is normal

(a) Urysohn Lemma (b) Tietze Characterisation (c) Extension theorem (d) Tychonoff theorem

15. Which of the following is not true

(a)  $T_4 \Rightarrow$  Completely regular (b)  $T_4 \Rightarrow$  Tychonoff (c)  $T_4 \Rightarrow T_3$  (d)  $T_4 \Rightarrow$  Second countable

16. Let  $A \subset X$  and  $f: A \rightarrow Y$  then the function  $F: X \rightarrow Y$  is the extension of  $f$  then which of the following is true

(a)  $F(a) = f(a) \forall a \in A$

(b)  $F(a) \neq f(a) \forall a \in A$

(c)  $F(a) = f(a)$  for some  $a \in A$

(d)  $F(a) \neq f(a)$ , for some  $a \in A$

17. Let  $A \subset X$  and  $f: A \rightarrow Y$  also  $F$  and  $G$  from  $X$  to  $R$  be two extensions of  $f$  then which of the following is true

(a)  $F(a) = G(a), \forall a \in \bar{A}$

(b)  $F(a) \neq G(a) \forall a \in \bar{A}$

(c)  $F(a) = G(a)$ , for some  $a \in A$

(d)  $F(a) \neq G(a)$ , for some  $a \in A$

18. Let  $A$  be a closed subset of the normal space  $X$  and suppose  $f: A \rightarrow [-1, 1]$  is a continuous function then there exist a continuous function  $F: X \rightarrow [-1, 1]$  such that  $F(x) = f(x), \forall x \in A$

Which of the following is related to the above statement .

(a) Urysohn Lemma (b) Tietze Extension theorem (c) Zones lemma (d) Embedding lemma

19. Let  $A$  be a closed subset of the normal space  $X$  and suppose  $f: A \rightarrow (-1, 1)$  is a continuous function then which of the following is true

(a) There exist a continuous function  $F: X \rightarrow (-1, 1)$  such that  $F(x) = f(x) \forall x \in A$

(b) There exist a continuous function  $F: X \rightarrow (-1, 1)$  such that  $F(x) \neq f(x) \forall x \in A$

(c) There exist a continuous function  $F: X \rightarrow (-1, 1)$  such that  $F(x) = 1, \forall x \in A$

(a) There exist a continuous function  $F: X \rightarrow (-1, 1)$  such that  $F(x) = -1 \forall x \in A$

20. Which of the following statements is true

- (1) Every regular Lindeloff space is normal
  - (2) Every regular Lindeloff space is second countable
  - (3) Every regular second countable space is normal
  - (4) Every regular second countable space is completely regular
- (a) 1 and 2      (b) 1 and 3      (c) 1 and 4      (d) All of the above

21. Which of the following statements are true

- (1) All  $T_4$  spaces are normal
  - (2) All  $T_4$  spaces are regular
  - (3) All  $T_4$  spaces are completely regular
  - (4) All  $T_4$  spaces are Hausdorff
- (a) 1 and 2      (b) 2 and 4      (c) 1,2 and 4      (d) All of the above

22. The range of a map from a compact space into a Hausdorff space is

- (a) Open      (b) Closed      (c) Quotient space of the domain      (d) Empty

23.  $X$  is a Hausdorff space and  $Y$  is a compact subset of  $X$  then  $Y$  is

- (a) Open      (b) Closed      (c) Clopen      (d) Empty

24. A topological space has the property that for every closed subset  $A$  of  $X$ , every continuous real valued function on  $A$  has a continuous extension to  $X$ , then  $X$  is

- (a) Compact      (b) Open      (c) Closed      (d) Normal

26. Let  $\{X_i: i \in I\}$  be an indexed family of sets then the cartesian product  $\prod_{i \in I} X_i$  is defined as

- (a) The set of all sequences  $(x_1, x_2, x_3, \dots, x_n, \dots)$  with  $x_i \in X_i$  for all  $i = 1, 2, \dots, n, \dots$
- (b) The set of all functions  $x$  from the index set  $I$  into  $\cup_{i \in I} X_i$  such that  $x(i) \in X_i$  for all  $i \in I$ .
- (c) The set of all functions  $x$  from the index set  $I$  into  $X_i$  such that  $x(i) \in X_i$  for all  $i \in I$ .
- (d) The set of all sequences in  $\cup_{i \in I} X_i$  such that  $x(i) \in X_i$  for all  $i \in I$ .

26. Let  $\{X_i: i \in I\}$  be an indexed family of sets and let  $X = \prod_{i \in I} X_i$ . Let  $J \in I$ , then the  $j$ th projection function  $\pi_j$  is defined as

- (a)  $\pi_j: X \rightarrow \cup_{i \in I} X_i$  defined by  $\pi_j(x) = x(j)$  for  $x \in X$ .
- (b)  $\pi_j: X \rightarrow \cup_{i \in I} X_i$  defined by  $\pi_j(x) = (x_1, x_2, x_3, \dots, x_n, \dots)$  for  $x \in X$ .
- (c)  $\pi_j: X \rightarrow X_j$  defined by  $\pi_j(x) = x(j)$  for  $x \in X$ .
- (d)  $\pi_j: X \rightarrow X_j$  defined by  $\pi_j(x) = x$  for  $x \in X$ .

27. Define a box in  $X = \prod_{i \in I} X_i$

- (a) A subset  $B$  of  $X$  of the form  $\bigcup_{i \in I} B_i$  for  $B_i \subset X_i, i \in I$ .
- (b) Any subset  $B$  of  $X$  is a box.
- (c) A subset  $B$  of  $X$  of the form  $\prod_{i \in I} B_i$  for  $B_i \subset X_i, i \in I$ .
- (d) A subset  $B$  of  $X$  of the form  $\prod_{i \in I} B_i$ .

28. Define a wall in  $X = \prod_{i \in I} X_i$

- (a) A set of the form  $\pi_j(B_j)$  for some  $j \in I$  and some  $B_j \subset X_j$ .
- (b) A set of the form  $\pi_j^{-1}(B_j)$  for some  $j \in I$  and some  $B_j \subset X_j$ .
- (c) A set  $B$  of  $X$  of the form  $\prod_{i \in I} B_i$ .
- (d) A set  $B$  of  $X$  of the form  $\bigcup_{i \in I} B_i$  for  $B_i \subset X_i, i \in I$ .

29. Which of the following is true

- a. A subset of  $X = \prod_{i \in I} X_i$  is a box iff it is the finite union of family of walls.
- b. A subset of  $X = \prod_{i \in I} X_i$  is a box iff it is the intersection of family of walls.
- c. A subset of  $X = \prod_{i \in I} X_i$  is a box iff it is the intersection of finitely many walls.
- d. A subset of  $X = \prod_{i \in I} X_i$  is a box iff it is the union of family of walls

30. Which of the following is true

- a. A subset of  $X = \prod_{i \in I} X_i$  is a large box iff it is the intersection of finitely many walls.
- b. A subset of  $X = \prod_{i \in I} X_i$  is a large box iff it is the intersection of family of walls.
- c. A subset of  $X = \prod_{i \in I} X_i$  is a large box iff it is the finite union of family of walls.
- d. A subset of  $X = \prod_{i \in I} X_i$  is a large box iff it is the union of family of walls.

31. For any sets  $Y, I$  and  $J$  which is true up to a set theoretic equivalence

- a.  $(Y^I)^J = Y^{I+J}$
- b.  $(Y^I)^J = Y^{I \cdot J}$
- c.  $(Y^I)^J = Y^{I \times J}$
- d.  $(Y^I)^J = Y^{I^J}$

32. Which of the following is true

- a. The intersection of family of walls is a large box.
- b. The intersection of any family of large boxes is a large box.
- c. The intersection of any family of boxes is a box.
- d. The union of family of walls is a box.

33. Let  $\{(X_i, \tau_i) : i \in I\}$  be an indexed collection of topological spaces and let  $X = \prod_{i \in I} X_i$  and for each  $i \in I$ ,  $\pi_i$  be the projection function. Then the product topology on  $X$  is

- a. The smallest topology on  $X$  which makes each projection function continuous.
- b. The topology on  $X$  which makes each projection function continuous.
- c. The largest topology on  $X$  which makes each projection function continuous.
- d. The strongest topology on  $X$  which makes each projection function continuous.

34. A large box is

- a. A box  $B = \prod_{i \in I} B_i$  where  $B_i = X_i, \forall i \in I$ .
- b. A box  $B = \prod_{i \in I} B_i$  where  $B_i \neq X_i$ , except for some  $i \in I$ .
- c. A box  $B = \prod_{i \in I} B_i$  where  $B_i = X_i$ , except for some finite  $i \in I$
- d. A box  $B = \prod_{i \in I} B_i$  where  $B_i \neq X_i, \forall i \in I$

35. A cube is

- a.  $[0,1]^I$ , where  $I$  is some set.
- b.  $[0,1]$
- c.  $[a,b] \times [c,d]$
- d.  $[0,1]^{[0,1]}$

36 . A Hilbert cube is

- a.  $[0,1]^I$ , where  $I$  is denumerable.
- b.  $[0,1]^I$ , where  $I$  is some set.
- c.  $[0,1]^H$
- d.  $[0,1]$

37 . Which of the following is true

- a. The projection function are continuous and open
- b. The projection function are continuous and closed
- c. The projection function are open and one to one
- d. The projection function are one to one and closed

38 . Let  $\tau$  be the product topology on the set  $X = \prod_{i \in I} X_i$ , where  $\{(X_i, \tau_i): i \in I\}$  is an indexed collection of topological spaces. Then standard sub base for the product topology  $\tau$  is

- The family of all subsets of the form  $\prod_{i \in I} V_i$  for  $V_i \in \tau_i, i \in I$ .
- The family of all subsets of the form  $\pi_i(V_i)$  for  $V_i \in \tau_i, i \in I$ .
- The family of all subsets of the form  $\pi_i^{-1}(V_i)$  for  $V_i \in \tau_i, i \in I$ .
- The family of all subsets of the form  $\prod_{i \in I} V_i$ .

39 . Let  $\tau$  be the product topology on the set  $X = \prod_{i \in I} X_i$ , where  $\{(X_i, \tau_i): i \in I\}$  is an indexed collection of topological spaces. Then standard base for the product topology  $\tau$  is

- The family of all walls all of whose sides are open in the respective spaces.
- The family of all boxes all of whose sides are open in the respective spaces.
- The family of all large boxes all of whose sides are open in the respective spaces.
- The family of all boxes in  $X$ .

40. If  $\{(X_i, \tau_i): i \in I\}$  is an indexed family of spaces having a topological property, the topological product  $\prod_{i \in I} X_i$  also has that property then that property is called a

- Productive property
- Countably Productive property.
- Finitely productive property
- None of these

41.  $T_0, T_1$ , and  $T_2$  are

- Productive properties
- Countably Productive properties
- Finitely productive properties
- None of these

42. Regularity and Complete regularity are

- Productive properties
- Countably Productive properties
- Finitely productive properties
- None of these

43. Tychonoff property is a

- Productive property
- Countably productive property.
- Finitely productive property
- None of these.

44. Connectedness is a

- a. Productive property
- b. Countably productive property
- c. Finitely productive property
- d. None of these

45. Metrisability is a

- a. Productive property
- b. Countably productive property
- c. Finitely productive property
- d. None of these

46. Cantor discontinuum is denoted by

- a.  $Z_2$
- b.  $Y^I$
- c.  $Z_2^I$
- d. None of these

47. Let  $S$  be a sub base for a topological space  $X$ . If for each  $V \in S$  and for each  $x \in V$ , there exist a continuous function  $f: X \rightarrow [0,1]$  such that  $f(x) = 0$  and  $f(y) = 1$  for all  $y$  not in  $V$ . Then

- a.  $X$  is regular
- b.  $X$  is Tychonoff
- c.  $X$  is completely regular.
- d. None of these

48. For any cardinal numbers  $\alpha, \beta, \gamma$ , which one is true

- a.  $(\alpha^\beta)^\gamma = \alpha^{\beta+\gamma}$
- b.  $(\alpha^\beta)^\gamma = \alpha^{\beta\gamma}$
- c.  $(\alpha^\beta)^\gamma = \alpha^{\beta-\gamma}$
- d. None of these

49. Let  $\{\alpha_i: i \in I\}$  be an indexed family of cardinal numbers. Then the cardinal number of the product set  $\prod_{i \in I} X_i$ , where for each  $i \in I$ ,  $X_i$  is a set of cardinality  $\alpha_i$  is

- a.  $\sum_{i \in I} \alpha_i$
- b.  $\prod_{i \in I} \alpha_i$
- c.  $\cup_{i \in I} \alpha_i$
- d. None of these

50. Consider the following two statements

(1) Each coordinate space of a topological product is completely regular then the product space is completely regular

(2) A product of topological spaces is completely regular then each coordinate space is completely regular

Which of the following is true

- (a) 1 is true      (b) 2 is true      (c) Both are true      (d) Both are false

51. Consider the following statements

(1) A topological product of spaces is Tychonoff then each coordinate space is Tychonoff

(2) Each coordinate space of a topological product is Tychonoff then the product space is not Tychonoff

Which of the following is true

- (a) 1 is true      (b) 2 is true      (c) Both are true      (d) Both are false

52. Choose the correct option from the following statements

(1) A topological product of spaces is connected then each coordinate space is not connected

(2) Each coordinate space of a topological product is connected then the product space is not connected

- (a) 1 is true      (b) 2 is true      (3) Both are true      (4) Both are false

53. Let  $\{Y_i: i \in I\}$  be an indexed family of sets. Suppose  $X$  is a set and let for each  $i \in I$ ,  $f_i: X \rightarrow Y_i$  be a function. Then the function  $e: X \rightarrow \prod_{i \in I} Y_i$  defined by  $e(x)(i) = f_i(x)$  for  $i \in I, x \in X$  is known as

- (a) Projection function      (b) Open function      (c) Evaluation function      (d) None of the above

54. An indexed family  $\{f_i: i \in I\}$  of functions all defined on the same domain  $X$  is said to distinguish points if

- (a) For any distinct  $x, y$  in  $X$  there exist  $j \in I$  such that  $f_j(x) \neq f_j(y)$   
(b) For any distinct  $x, y$  in  $X$  there exist  $j \in I$  such that  $f_j(x) = f_j(y)$   
(c) For any distinct  $x, y$  in  $X$  there exist  $j \in I$  such that  $f_j(x) > f_j(y)$   
(d) For any distinct  $x, y$  in  $X$  there exist  $j \in I$  such that  $f_j(x) < f_j(y)$

55. The evaluation function of a family of function is one-to-one if and only if

- (a) That family do not distinguish points
- (b) That family distinguishes points
- (c) That family distinguish points from closed sets
- (d) That family do not distinguish points from closed sets

56. An indexed family of functions  $\{f_i: X \rightarrow Y_i: i \in I\}$ , where  $X, Y_i$  are topological spaces, is said to distinguish points from closed sets in  $X$  if

- (a) For any  $x \in X$  and any closed subset  $C$  of  $X$  not containing  $x$  there exist  $j \in I$  such that  $f_j(x) \in \overline{f_j(C)}$  in  $Y_j$ .
- (b) For any  $x \in X$  and any closed subset  $C$  of  $X$  containing  $x$  there exist  $j \in I$  such that  $f_j(x) \in \overline{f_j(C)}$  in  $Y_j$ .
- (c) For any  $x \in X$  and any closed subset  $C$  of  $X$  not containing  $x$  there exist  $j \in I$  such that  $f_j(x) \notin \overline{f_j(C)}$  in  $Y_j$ .
- (d) For any  $x \in X$  and any closed subset  $C$  of  $X$  containing  $x$  there exist  $j \in I$  such that  $f_j(x) \notin \overline{f_j(C)}$  in  $Y_j$ .

57. If the family of all continuous real valued functions on a topological space  $\tau$  distinguish points from closed sets then which among the following is necessarily true?

- (a)  $\tau$  is not completely regular
- (b)  $\tau$  is completely regular
- (c)  $\tau$  is normal
- (d)  $\tau$  is not normal.

58. Let  $\{f_i: X \rightarrow Y_i: i \in I\}$  be a family of functions which distinguishes points from closed sets in  $X$ . Then the corresponding evaluation function  $e: X \rightarrow \prod_{i \in I} Y_i$  is

- (a) Open when regarded as a function from  $X$  onto  $e(X)$
- (b) Not necessarily open when regarded as a function from  $X$  onto  $e(X)$
- (c) Closed when regarded as a function from  $X$  onto  $e(X)$
- (d) Neither open nor closed when regarded as a function from  $X$  onto  $e(X)$ .

59. Let  $\{f_i: X \rightarrow Y_i: i \in I\}$  be a family of continuous functions. Then the corresponding evaluation map is an embedding of  $X$  into the product space  $\prod_{i \in I} Y_i$  if the family

- (a) Distinguish points only
- (b) Distinguish points from closed sets only
- (c) Distinguish points and also distinguish points from closed sets
- (d) Neither distinguish points nor distinguish points from closed sets

60. The embedding theorem states that

- (a) A topological space is completely regular iff it is embeddable into a cube.
- (b) A topological space is Tychonoff iff it is embeddable into a cube.
- (c) A topological space is regular iff it is embeddable into a cube.
- (d) A topological space is completely normal iff it is embeddable into a cube.

61. A space is embeddable in the Hilbert cube if and only if

- (a) It is second countable      (b) It is  $T_3$       (c) It is second countable and  $T_3$
- (d) It is neither second countable nor  $T_3$

62. A second countable space is metrisable if and only if

- (a) It is  $T_0$       (b) It is  $T_1$       (c) It is  $T_2$       (d) It is  $T_3$

63. Urysohn's metrisation theorem states that

- (a) A second countable space is metrisable iff it is  $T_0$
- (b) A second countable space is metrisable iff it is  $T_1$
- (c) A second countable space is metrisable iff it is  $T_2$
- (d) A second countable space is metrisable iff it is  $T_3$

64. Let  $X$  be a topological space. Then a family  $\mathcal{U}$  of subsets of  $X$  is said to be locally finite if for each  $x \in X$ , there exists a neighbourhood  $N$  of  $x$  which intersects

- (a) Only finitely many members of  $\mathcal{U}$       (b) Infinitely many members of  $\mathcal{U}$
- (c) No members of  $\mathcal{U}$       (d) Either A or B

65. Let  $X$  be a topological space. Then a family  $\mathcal{V}$  of subsets of  $X$  is said to be  $\sigma$ -locally finite if it can be written as the union of

- (a) Countably many subfamilies each of which is finite
- (b) Countably many subfamilies each of which is locally finite
- (c) Uncountably many subfamilies each of which is finite
- (d) Uncountably many subfamilies each of which is locally finite

66. A topological space is said to be countably compact if

- (a) Every open cover of it has a finite sub-cover
- (b) Every countable open cover of it has a finite sub-cover
- (c) Every open cover of it has a countable subcover
- (d) None of the above

67. Which among the following statements is not correct?

- (a) A space is compact if and only if it is countably compact and Lindeloff.
- (b) A space is countably compact if every countable open cover of it has a finite sub-cover.
- (c) A continuous image of a countable compact space is countably compact.
- (d) Countable compactness is not a weakly hereditary property.

68. For a  $T_1$  countably compact topological space  $X$  which among the following is/are true?

- (a) Every countable family of closed subsets of  $X$  which has the finite intersection property has non-empty intersection
- (b) Every infinite subset of  $X$  has an accumulation point
- (c) Every sequence in  $X$  has a cluster point
- (d) All the above

69. For a  $T_1$  countably compact topological space  $X$  which among the following is/are true?

(a) Every countable family of closed subsets of  $X$  which has the finite intersection property has non-empty intersection

(b) Every infinite subset of  $X$  has an accumulation point

(c) Every infinite open cover of  $X$  has a proper sub-cover

(d) All the above

70. Let  $X$  be a  $T_1$  topological space such that every sequence in  $X$  has a cluster point. Then which among the following is/are true?

(a)  $X$  is countably compact

(b) Every infinite subset of  $X$  has an accumulation point

(c) Every infinite open cover of  $X$  has a proper sub-cover

(d) All the above

71. Consider the following two statements.

- i. A continuous image of a countably compact space is countably compact.
- ii. Countable compactness is a weakly hereditary property.

Then choose the correct option.

(a) Both i. and ii. are true      (b) i. is true but ii. is false

(c) ii. is true but i. is false      (d) both i. and ii. are false

72. Consider the following two statements.

- i. Countable compactness is not a weakly hereditary property
- ii. If in a  $T_1$  countably compact topological space  $X$  every sequence has a cluster point then  $X$  is countably compact

Then choose the correct option.

(a) Both i. and ii. are true      (b) i. is true but ii. is false

(c) ii. is true but i. is false      (d) both i. and ii. are false

73. Consider the following two statements.

- i. A metric space is compact if and only if it is countably compact
- ii. Every countably compact metric space is second countable

Then choose the correct option.

- (a) Both i. and ii. are true
- (b) i. is true but ii. is false
- (c) ii. is true but i. is false
- (d) both i. and ii. are false

74. Consider the following two statements.

- i. Countable compactness is not a weakly hereditary property
- ii. If in a  $T_1$  topological space  $X$  every sequence has a cluster point then  $X$  is countably compact

Then choose the correct option.

- (a) Both i. and ii. are true
- (b) i. is true but ii. is false
- (c) ii. is true but i. is false
- (d) both i. and ii. are false

75. Consider the following two statements.

- i. A metric space is compact if and only if it is countably compact
- ii. Every continuous real valued function on a countably compact space is bounded and attains its extrema

Then choose the correct option.

- (a) Both i. and ii. are true
- (b) i. is true but ii. is false
- (c) ii. is true but i. is false
- (d) both i. and ii. are false

76. Consider the following two statements.

- i. A space is sequentially compact if every sequence in it has a convergent subsequence
- ii. A first countable, countably compact space need not be sequentially compact

Then choose the correct option.

- (a) Both i. and ii. are true
- (b) i. is true but ii. is false
- (c) ii. is true but i. is false
- (d) both i. and ii. are false

77. Let  $X$  be second countable space. Then choose the right option.

- (a)  $X$  is compact if and only if it is countably compact
- (b)  $X$  is countably compact if and only if it is sequentially compact
- (c)  $X$  is compact if and only if it is sequentially compact
- (d) All the above

78.. Consider the following statements

- (i) A countably compact metric space is second countable
- (ii) A countably compact metric space is compact

Then choose the correct option

- (a) Both i and ii are true
- (b) i is true but ii is false
- (c) ii is true but i is false
- (d) Both i and ii are false

79.  $f$  is a continuous function from  $X$  to  $\mathbb{R}$  where  $X$  is countably compact then  $f$  is

- (a) Open
- (b) Closed
- (c) Bounded
- (d) Unbounded

80 . Which of the following statement is not true

- (a) Continuous image of a countably compact space is countably compact
- (b) Countable compactness is a hereditary property
- (c) Countable compactness is a weakly hereditary property
- (d) Acountable compact metric space is second countable

81. A sequence in a set  $X$  is a function;
- $f: \mathbb{N} \rightarrow X$ , where  $\mathbb{N}$  is the set of all natural numbers
  - $f: D \rightarrow X$ , where  $D$  is a directed set
  - $f: X \rightarrow \mathbb{N}$ , where  $\mathbb{N}$  is the set of all natural numbers
  - $f: X \rightarrow D$ , where  $D$  is a directed set
82. A directed set is a pair  $(D, \geq)$  where  $D$  is a non-empty set and  $\geq$  a binary relation on  $D$  satisfying;
- (i) Transitivity (ii) Reflexivity (iii) Symmetry
  - (i) Transitivity (ii) Reflexivity (iii) Anti-symmetry
  - (i) Transitivity (ii) Reflexivity (iii) For all  $m, n \in D$ , there exists  $p \in D$  such that  $p \geq m$  and  $p \geq n$
  - (i) Transitivity (ii) Anti-symmetry (iii) For all  $m, n \in D$ , there exists  $p \in D$  such that  $m \geq p$  and  $n \geq p$
83. A net in a set  $X$  is a function;
- $S: X \rightarrow D$  where  $D$  is a directed set
  - $S: D \rightarrow X$  where  $D$  is a partially ordered set
  - $S: X \rightarrow D$  where  $D$  is a totally ordered set
  - $S: D \rightarrow X$  where  $D$  is a directed set
84. Which of the following is not an example of a net;
- $(\mathbb{N}, \geq)$  where  $\mathbb{N}$  is the set of all natural numbers and  $\geq$  is the usual ordering on  $\mathbb{N}$
  - $(\eta_x, \geq)$  where  $\eta_x$  is the neighbourhood system at  $x$  and for  $U, V \in \eta_x$ ,  $U \geq V$  iff  $V \subset U$
  - $(D, \geq)$  where  $D$  is the family of all open neighbourhoods of  $x$  and for  $U, V \in D$ ,  $U \geq V$  iff  $U \subset V$
  - $(D, \geq)$  where  $D = \eta_x \times \eta_y$  and for  $(U_1, V_1), (U_2, V_2) \in D$ ,  $(U_1, V_1) \geq (U_2, V_2)$  iff  $U_1 \subset U_2$  and  $V_1 \subset V_2$
85. Which of the following is true;
- A net is a function with codomain is a directed set
  - A net is a function with domain is a directed set
  - Every net converges to some point
  - Every net has a cluster point
86. For a topological space  $X$ , the limits of all nets in  $X$  are unique. Then;
- $X$  is a Hausdorff space
  - $X$  is not a Hausdorff space
  - There exist  $x, y \in X$  such that  $U \cap V \neq \phi$  for all  $U \in \eta_x$  and  $V \in \eta_y$
  - For any distinct  $x, y \in X$  we have  $U \cap V \neq \phi$  for all  $U \in \eta_x$  and  $V \in \eta_y$
87.  $D$  is a directed set and  $E$  is an eventual subset of  $D$ , then;
- For any  $m \in D$  there exists  $n \in D$  such that  $n \geq m$  and  $n \notin E$
  - Every element of  $D$  is in  $E$
  - There exists  $m \in D$  such that for all  $n \in D$ ,  $n \geq m$  implies that  $n \in E$
  - There exists  $m \in D$  such that for all  $n \in D$ ,  $m \geq n$  implies that  $n \in E$

88.  $S: D \rightarrow X$  is a net in  $X$  and  $S$  is eventually in a subset  $A$  of  $X$ , then;
- $A$  contains all the terms of  $S$  after a certain stage
  - $A$  contains all the terms of  $S$
  - $A$  contains no terms of  $S$
  - For every  $m \in D$  there exists  $n \in D$  such that for all  $n \geq m$  and  $S_n \notin A$
89. A subset  $E$  of a directed set  $D$  has the property that for every  $m \in D$ , there exists  $n \in E$  such that  $n \geq m$ , then;
- $E$  is an eventual subset of  $D$
  - $E$  is a cofinal subset of  $D$
  - None of the above is true
  - Both (a) and (b) are true
90. A net  $S: D \rightarrow X$  is said to be frequently in a subset  $A$  of  $X$  if;
- $S^{-1}(A)$  is a cofinal subset of  $D$
  - $S^{-1}(A)$  is an eventual subset of  $D$
  - $S$  converges to a point in  $X$
  - None of the above is true
91. If a net  $S$  converges to  $x$ , then for every neighbourhood  $U$  of  $x$ ;
- $S$  is eventually in  $U$
  - $S$  is not eventually in  $U$
  - $S$  is not frequently in  $U$
  - None of the above is true
92. If a net  $S$  converges to  $x$ , then which of the following are true;
- $x$  is a cluster point of  $S$
  - $S$  is eventually in  $U$  for every neighbourhood  $U$  of  $x$
  - $S$  is frequently in  $U$  for every neighbourhood  $U$  of  $x$
- (1) only is true
  - Only (1) and (2) are true
  - None is true
  - All are true
93. If  $f$  is homotopic to  $f'$  where  $f$  is a continuous map and  $f'$  is a constant map then;
- $f$  is a path
  - $f$  is null-homotopic
  - $f$  is path homotopic
  - None of the above
94.  $f: [0, 1] \rightarrow X$  is a continuous function such that  $f(0) = x_0$  and  $f(1) = x_1$ , then;
- $f$  is a constant path in  $X$
  - $f$  is a path in  $X$  from  $x_1$  to  $x_0$
  - $f$  is a path in  $X$  from  $x_0$  to  $x_1$

(d)  $f$  is a path in  $[0, 1]$

95. . The relation homotopy is;

- (a) Reflexive but not symmetric
- (b) Reflexive and symmetric but not transitive
- (c) Not reflexive
- (d) Reflexive, symmetric and transitive

96. The relation path homotopy is;

- (a) Not reflexive
- (b) Not symmetric
- (c) Both (a) and (b) holds
- (d) None is true

97.  $f$  is a path in  $X$  from  $x_0$  to  $x_1$  and  $g$  is a path in  $X$  from  $x_1$  to  $x_2$ , then  $f * g$  is;

- (a) A path in  $X$  from  $x_0$  to  $x_1$
- (b) A path in  $X$  from  $x_0$  to  $x_2$
- (c) A path in  $X$  from  $x_2$  to  $x_0$
- (d) A path in  $X$  from  $x_1$  to  $x_2$

98.  $f$  and  $g$  are any two maps of a space  $X$  into  $\mathbb{R}^2$ , then;

- (a)  $f$  and  $g$  are homotopic
- (b)  $f$  and  $g$  are not homotopic
- (c)  $f$  and  $g$  are paths in  $X$
- (d) None of the above

99.  $f$  and  $g$  are paths in  $X$ , then the product  $f * g$  is a path in  $X$  if;

- (a)  $f$  and  $g$  have the same initial point and the same final point
- (b) Final point of  $f$  is the initial point of  $g$
- (c) Initial point of  $f$  is the initial point of  $g$
- (d) Initial point of  $f$  is the final point of  $g$

100.  $F$  is a path homotopy between  $f$  and  $f'$  then for any  $s \in I$ ;

- (a)  $F(s, 0) = f'(s)$
- (b)  $F(s, 0) = x_0$
- (c)  $F(s, 0) = f(s)$
- (d)  $F(s, 0) = x_1$

