Real Analysis- First Semester

1. Which of the following functions is monotonically decreasing when $x > 0$?
    A. $2x + 3$
    B. e^x
    C. $\frac{1}{x}$
    D. $logx$
2. Which of the following functions is monotonically increasing in the interval $(-\infty, \infty)$?
    A. x^3
    B. $\frac{1}{x}$
    C. $sinx$
    D. $|x|$
3. Strictly increasing function is defined as
    A. If $x_1 < x_2$ then $f(x_1) = f(x_2)$
    B. If $x_1 < x_2$ then $f(x_1) < f(x_2)$
    C. If $x_1 < x_2$ then $f(x_1) > f(x_2)$
    D. None of the above
4. Strictly decreasing function is defined as
    A. If $x_1 < x_2$ then $f(x_1) = f(x_2)$
    B. If $x_1 < x_2$ then $f(x_1) < f(x_2)$
    C. If $x_1 < x_2$ then $f(x_1) > f(x_2)$
    D. None of the above
5. Let f be a real valued function defined on $[a, b]$ and $c \in (a, b)$ then c is called a jump of discontinuity of f if
    A. $f(c+) - f(c-) > 0$
    B. $f(c+) - f(c-) = 0$
    C. $f(c) \quad - f(c-) = 0$
    D. $f(c+) - f(c) \quad = 0$
6. Find the point of discontinuity of the function $f(x) = \frac{1}{x-2}$.
    A. -2
    B. 0
    C. 2
    D. 1
7. Which of the following is not a partition of [0,1].
    A. {0, 0.2, 0.4, 0.6, 0.8}
    B. {0, 0.3, 06 ,0.7, 1}
    C. {0, 0.3, 0.6, 0.9, 1}
    D. {0, 0.5,1})
8. Find the length of the second subinterval of the partition {1, 1.3, 1.5, 1.9, 2}.
    A. 0.1
    B. 0.2
    C. 0.3
    D. 0.4
9. Which of the following functions are bounded?
    A. x
    B. $\frac{1}{x}$

C. $\frac{1}{1+x^{2},}$
D. $tanx$

10. Assume that f and g are each of bounded variation on $[a, b]$. Which of the following function need not be of bounded variation?
    A. $f + g$
    B. $f - g$
    C. $f * g$
    D. $f \div g$
11. If f is monotonic function on $[a, b]$
    A. f is bounded variation
    B. f is unbounded
    C. The set of discontinuities of f are uncountable
    D. None of the above
12. A function of bounded variation is
    A. Necessarily bounded
    B. Necessarily unbounded
    C. May be bounded or unbounded
    D. None of the above
13. f is of bounded variation on $[a, b]$ if and only if
    A. f is the difference of two increasing real valued functions on $[a, b]$
    B. f is the product of two increasing real valued functions on $[a, b]$
    C. f is the quotient of two increasing real valued functions on $[a, b]$
    D. None of the above
14. If f is bounded variation on $[a, b]$ Then total variation on $[a, b]$ is
    A. Non negative finite number
    B. Non positive finite number
    C. Extended real number
    D. None of the above
15. The total variation of f on $[a, b]$ is 0 then
    A. f is continuous
    B. f is constant
    C. f is monotonic
    D. None of the above
16. Assume that f and g are each of bounded variation on $[a, b]$. Then
    A. $v_{f+g} = v_f + v_g$
    B. $v_{f+g} \leq v_f + v_g$
    C. $v_{f+g} \geq v_f + v_g$
    D. None of the above
17. Assume that f and g are each of bounded variation on $[a, b]$. Then
    A. $v_{f-g} = v_f + v_g$
    B. $v_{f-g} \leq v_f + v_g$
    C. $v_{f-g} \geq v_f + v_g$
    D. None of the above
18. Assume that f and g are each of bounded variation on $[a, b]$. Then
    A. $v_{f.g} = A.v_f + B.v_g$
    B. $v_{f.g} \leq A.v_f + B.v_g$
    C. $v_{f.g} \geq A.v_f + B.v_g$
    D. None of the above
19. If f is bounded variation on $[a, b]$ and $c \in (a, b)$, Then

A. $v_f(a,b) \le v_f(a,c) + v_f(c,b)$
B. $v_f(a,b) \ge v_f(a,c) + v_f(c,b)$
C. $v_f(a,b) = v_f(a,c) + v_f(c,b)$
D. None of the above

20. Which of the following function trace out the unit circle $x^2 + y^2 = 1$
A. $e^{-2\pi i}$
B. $sin\pi z$
C. $cos\pi z$
D. $tan\pi z$

21. The length of any inscribed polygon is --------- that of the curve.
A. Greater than or equal to
B. Less than or equal to
C. Strictly Greater than
D. Equal to

22. Let $\vec{f}: [a,b] \to R^n$ be a path in R^n. Then its arc length
A. $\Lambda_{\vec{f}}(a,b) = Sup\{\Lambda_{\vec{f}}(P): P \in \mathcal{P}[a,b]\}$
B. $\Lambda_{\vec{f}}(a,b) = Inf\left\{\Lambda_{\vec{f}}(P): P \in \mathcal{P}[a,b]\right\}$
C. $\Lambda_{\vec{f}}(a,b) = Min\left\{\Lambda_{\vec{f}}(P): P \in \mathcal{P}[a,b]\right\}$
D. None of the above

23. Let $\vec{f}: [a,b] \to R^n$ be a path in R^n with components $\vec{f} = (f_1, f_2, \dots, f_n)$. Then
A. $\Lambda_{\vec{f}}(a,b) \le v_1(a,b) + v_2(a,b) + \cdots + v_n(a,b)$
B. $\Lambda_{\vec{f}}(a,b) \ge v_1(a,b) + v_2(a,b) + \cdots + v_n(a,b)$
C. $\Lambda_{\vec{f}}(a,b) = v_1(a,b) + v_2(a,b) + \cdots + v_n(a,b)$
D. None of the above

24. Let $\vec{f} = (f_1, f_2, \dots, f_n)$ be a rectifiable path defined on $[a,b]$ and $c \in (a,b)$. Then
A. $\Lambda_{\vec{f}}(a,b) \le \Lambda_{\vec{f}}(a,c) + \Lambda_{\vec{f}}(c,b)$
B. $\Lambda_{\vec{f}}(a,b) \ge \Lambda_{\vec{f}}(a,c) + \Lambda_{\vec{f}}(c,b)$
C. $\Lambda_{\vec{f}}(a,b) = \Lambda_{\vec{f}}(a,c) + \Lambda_{\vec{f}}(c,b)$
D. None of the above

25. Let $\vec{f}: [a,b] \to R^n$ and $\vec{g}: [c,d] \to R^n$ be two paths in R^n, each of which is one to one on its domain. Let $u: [c,d] \to [a,b]$ be the real valued function satisfying $\vec{g}(t) = \vec{f}(u(t))$. Then
A. u is continuous and strictly monotonic
B. u is continuous and monotonic
C. u is monotonic
D. None of the above

26. Let $I = [3,12]$ be a closed and bounded interval in R. let $P_1 = (3,5,9,12)$ and $P_2 = (3,4,5,7,9,11,12)$, $P_3 = (3,9,12)$ be any three partitions of I then

A) P_2 is a refinement of P_1 and P_3 is a refinement of P_1
B) P_1 is a refinement of P_2 and P_2 is a refinement of P_3
C) P_2 is a refinement of P_3 and P_1 is a refinement of P_3
D) P_2 is a refinement of P_1 and P_1 is a refinement of P_3

27. Upper Riemann Stieltjes integral of f with respect to α over $[a,b]$ is

A) $Sup\,\{L(P,f,\alpha)\}$

B) $Inf\{L(P,f,\alpha)\}$
C) $Inf\{U(P,f,\alpha)\}$
D) $Sup\{U(P,f,\alpha)\}$

28. Let $I = [1,13]$ be closed and bounded interval in R. Let $P = (1,2,5,9,12,13)$ be any

partition of I, then $||P||$ is

A) 3
B) 4
C) 2
D) 1

29. Let $f(x)$ be a bounded real valued function defined on $[a,b]$ and $\alpha(x)$ is monotonicallty

increasing on $[a,b]$.Let P be any partition of $[a,b]$ .let P be any partition of $[a,b]$ then

A) $L(P,-f,\alpha) = L(P,f,\alpha)$
B) $U(P,-f,\alpha) = -U(P,f,\alpha)$
C) $L(P,-f,\alpha) = -U(P,f,\alpha)$
D) $L(P,-f,\alpha) = U(P,f,\alpha)$

30. Let $I = [2,8]$ be closed and bounded interval in R. Let $P_1 = (2,5,7,8)$ and

$P_2 = (2,4,6,7,8)$ then common refinement of P_1 and P_2 is

A) (2,7,8)
B) (2,4,6,7,8)
C) (2,4,6,6,7,8)
D) (2,5,7,8)

31. Let f be a bounded function and α be non decreasing function on $[a,b]$ then

A) $\int_{-a}^{b} f d\alpha = \int_{a}^{-b} f d\alpha$
B) $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha$
C) $\int_{-a}^{b} f d\alpha \geq \int_{a}^{-b} f d\alpha$
D) $\int_{-a}^{b} f d\alpha = -\int_{a}^{-b} f d\alpha$

32 If f is continuous function on $[a,b]$ then
A) f is differentiable on $[a,b]$
B) f is non-differentiable on $[a,b]$
C) f is Reimann Steiljets integrable on $[a,b]$
D) f is not Reimann Steiljets integrable on $[a,b]$

33 If P is refinement of Q then
A) $L(P,f,\alpha \leq L(Q,f,\alpha)$
B) $U(P,f,\alpha) \leq U(Q,f,\alpha)$
C) $L(Q,f,\alpha) \leq L(P,f,\alpha)$

D) $L(P, f, \alpha) \leq U(Q, f, \alpha)$

34 Let f $\epsilon R(\alpha)$ then $\alpha(x)$ can be

A) $\frac{1}{x}$

B) $\frac{1}{1+x^2}$

C) x^2

D) None of the above

35 Let f and g two bounded functions defined on $[a, b]$, p be any partition of $[a, b]$ then

A) $L(P, f+g) \geq L(P, f) + L(P, g)$

B) $L(P, f+g) \leq L(P, f) + L(P, g)$

C) $L(P, f+g) = L(P, f) + l(P, g)$

D) $L(P, f+g) = 2[L(P, f) + l(P, g)]$

36 Lower Riemann Stieltjes integral of f with respect to α over $[a, b]$ is

A) $Sup\{L(P, f, \alpha)\}$

B) $Inf\{L(P, f, \alpha)\}$

C) $Inf\{U(P, f, \alpha)\}$

D) $Sup\{U(P, f, \alpha)\}$

37 Let $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a, b]$ then $\int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$ is

A) $\int_a^b f_1 + f_2 \, d\alpha$

B) 2 $\int_a^b f_1 + f_2 \, d\alpha$

C)) $\int_a^b f_1 f_2 \, d\alpha$

D) 0

38 Consider the function $f(x) = x^2 + 3$. Which of the following is correct

A) Riemann integrable on [0,2]

B) Riemann integrable on [0,1]

C) Riemann integrable on [0,5]

D) All are correct

39 Find the value of $\int_0^1 x d\alpha(x)$

A) $\frac{1}{2}$

B) $\frac{2}{3}$

C) $\frac{1}{2}$

D) $-\frac{1}{2}$

40 Lower Riemann Stieltjes integral of f with respect to α over $[a, b]$ is

A) $Sup\{L(P, f, \alpha)\}$

B) $Inf\{L(P, f, \alpha)\}$

C) $Inf\{U(P, f, \alpha)\}$

D) $Sup\{U(P, f, \alpha)\}$

41 Which of the following is true

A) $\int_a^b f + g \, d\alpha \leq \int_a^b f \, d\alpha + \int_a^b g \, d\alpha$
B) $\int_a^b f + g \, d\alpha = \int_a^b f \, d\alpha + \int_a^b g \, d\alpha$
C) $\int_a^b f + g \, d\alpha \geq \int_a^b f \, d\alpha + \int_a^b g \, d\alpha$
D) $\int_a^b f + g \, d\alpha = \int_a^b f \, d\alpha$

42 If $f_1(x) \leq f_2(x)$ on $[a, b]$ then

A) $\int_a^b f_1 \, d\alpha \leq \int_a^b f_2 d\alpha$
B) $\int_a^b f_1 \, d\alpha = \int_a^b f_2 d\alpha$
C) $\int_a^b f_1 \, d\alpha \geq \int_a^b f_2 d\alpha$
D) $\int_a^b f_1 \, d\alpha = -\int_a^b f_2 d\alpha$

43 If $f \in R(\alpha)$ on $[a, b]$ and c be any positive integer

A) $cf \in R(\alpha) \; and \; f \notin R(c\alpha)$
B) $cf \in R(\alpha) \; and \; f \in R(c\alpha)$
C) $cf \notin R(\alpha) \; and \; f \notin R(c\alpha)$
D) $cf \notin R(\alpha) \; and \; f \in R(c\alpha)$

44 Let f be a bounded function and α is nondecreasing function on $[a, b]$ then

A) $\int_{-a}^{b} (-f) \, d\alpha = -\int_{-a}^{b} f \, d\alpha$
B) $\int_{-a}^{b} (-f) \, d\alpha = -\int_{a}^{-b} f \, d\alpha$
C) $\int_{-a}^{b} (-f) \, d\alpha = \int_{-a}^{b} f \, d\alpha$
D) $\int_{-a}^{b} (-f) \, d\alpha = \int_{a}^{-b} f \, d\alpha$

45 Let $\alpha(x) = x \;\; \forall x \in [a, b]$ be a monotonic increasing function, then $\sum_{i=1}^{n} \Delta\alpha_i$ is

A) $b - a$
B) $a + b$
C) $a - b$
D) 0

46 If $f \in R(\alpha)$ and if there is differentiable function F on $[a, b]$ such that $F' = f$ then $\int_a^b f(x) \, dx$ is

A) $F(b) + F(a)$
B) $F(b) - F(a)$
C) $F(a) - F(b)$
D) 0

47 Let $f(x) = 2$, $x \in [1,100]$ then $f(x)$ is

A) $f(x)$ is Riemann integrable in $[1,100]$

B) $f(x)$ is not Riemann integrable in $[1,100]$
C) $f(x)$ may or may not be Riemann integrable in $[1,100]$
D) None of the above

48 Let P' be a refinement of partition P, $U(P,f,\alpha) - L(P,f,\alpha) < \epsilon$ for some $\epsilon > 0$ $then$

A) $U(P',f,\alpha) - L(P',f,\alpha) > \epsilon$
B) $U(P',f,\alpha) - L(P',f,\alpha) < \epsilon$
C) $U(P',f,\alpha) + L(P',f,\alpha) > \epsilon$
D) $U(P',f,\alpha) + L(P',f,\alpha) < \epsilon$

49 If f be bounded in [a,b] and c be a constant, then evaluate $\int_a^b f\, dc$

A) 0
B) 1
C) $\frac{1}{2}$
D) -1

50 Let f be a bounded function and α be non decreasing function on $[a,b]$ then

Which of the following is true

A) $m\,[\alpha(b) - \alpha(a)] \leq L(P,f,\alpha)$
B) $M\,[\alpha(b) - \alpha(a)] \leq L(P,f,\alpha)$
C) $m\,[\alpha(b) - \alpha(a)] \leq U(P,f,\alpha)$
D) None of the above

51. Which one of the following statements are correct

A) pointwise convergence → uniform convergence
B) uniform convergence → pointwise convergence
C) uniform convergence → uniform continuity
D) All of the above

52. Suppose $\{f_n\}$ be a sequence of functions such that $|f_n(x)| \leq M_n$, $n = 1,2,....$ If $\sum M_n$ converges then

$A)$ $\sum f_n$ converges

$B)$ $\sum f_n$ converges uniformly

$C)$ $\sum f_n$ diverges

$D)$ $\sum f_n$ converges to a continuous and bounded function f

53. Suppose $\lim_{n\to\infty} f_n \quad (x) = f(x)$ and $M_n = Sup|f_n(x) - f(x)|$ Then

(A) $f_n \to f$ if $M_n \to \infty$ as $n \to 0$
(B) $f_n \to f$ uniformly if and only if $M_n \to 0$ as $n \to \infty$
(C) f_n diverges
(D) $f_n \to f$ if $M_n \to \infty$ as $n \to 0$

54. Let x be a limit point of a set E. $\{f_n\}$ be a sequence of functions defined on the set E. Then $\lim_{t\to x}\lim_{n\to\infty} f_n(t) = \lim_{n\to\infty}\lim_{t\to x} f_n(t)$ if

(A) $\{f_n\}$ and f are continuous on E
(B) $\{f_n\}$ cconverges to f on E
(C) $\{f_n\}$ converges uniformly on E
(D) $\{f_n\}$ converges to f on E and f is continuous on E

55.Which of the following statements are correct

(A) $\{f_n\}$ is a sequence of continuous functions , $f_n \to f$ uniformly then f is continuous
(B) $\{f_n\}$ is a sequence of continuous function and $f_n \to f$, where f is a continuous function. Then the convergence is uniform.
(C) $\{f_n\}$ converges to a continuous function f. Then the convergence is uniform.
(D) None of the above

56. Let a sequence of continuous functions $\{f_n\}$ converges pointwise to a continuous function f on a set K such that $f_n(x) \geq f_{n+1}(x)\ \forall\ x \epsilon\ K$. Then the convergence is uniform if

(A) K is open
(B) K is closed
(C) K is compact
(D) K is bounded

57. Which one of the following statements are correct

(A) f is continuous $\to$ f is differentiable
(B) f is differentiable $\to$ f is continuous
(C) Both (A) and (B)
(D) Neither (A) nor (B)

58. Which one of the following sequence of functions converges uniformly

(A) $f_n(x)=\frac{1}{nx+1}$; $x \in (0,1)$
(B) $f_n(x)=\frac{1}{nx+1}$; $x \in [0,1]$
(C) $f_n(x)=\frac{1}{nx+1}$; $x \in (0,1]$
(D) All of the above

59. $\{f_n\}$ is a sequence of continuous functions , $f_n \to f$ uniformly then

(A) f is continuous
(B) f is differentiable
(C) f is bounded
(D) f is equicontinuous

60. A series of functions $\sum f_n$ converges uniformly to a function f on a set E if

(A) $\{f_n\}$ converges to f on E
(B) Sequence of partial sums converges uniformly on E
(C) Each f_n is continuous and $\{f_n\}$ converges to f on E
(D) None of the above

61. Consider the sequences of functions $\{f_n\} = \frac{1}{nx+1}$; $x \in (a,b)$. Then

(A) $\{f_n\}$ is sequence of continuous functions

(B) $f_n \to 0$ monotonically
(C) $\{f_n\}$ converges uniformly
(D) Both (A) and (B)

62. Let $|f_n| \leq M_n; \quad n = 1,2,3 \ldots.$ Then the series $\{f_n\}$ converges uniformly if

(A) $\sum M_n \to 0$
(B) $\sum M_n \to 0$ uniformly
(C) $\sum M_n$ converges
(D) $\sum M_n$ converges uniformly

63. $\{f_n\}$ is sequence of continuous functions on a compact set K. $\{f_n\} \to f$ pointwise to a continuous function f on K and $f_{n+1}(x) \leq f_n(x) \ \forall x \in K$. Let $g_n = f_n - f$. Then

(A) g_n is continuous
(B) $\{g_n\} \to 0$ pointwise
(C) $g_{n+1}(x) \leq g_n(x) \ \forall x \in K$
(D) All of the above

64. Let $C(x)$ be the set of all complex valued, continuous, bounded functions on X. Then the supremum norm is defined as

(A) $||f|| = \sup_{x \in X} |f(x)|$
(B) $||f|| = \sup_{x \in X} f(x)$
(C) $||f|| = \sup_{x \in X} f'(x)$
(E) None of the above

65. Let $C(x)$ be the set of all complex valued, continuous, bounded functions on X. Define $||f|| = \sup_{x \in X} |f(x)|$. Which one of the following is correct

(A) $||f|| < \infty$
(B) $||f|| = 0$ if and only if $f(x) = 0 \ \forall x \in X$
(C) $||f + g|| \leq ||f|| + ||g||$
(D) All of the above

66. Let X be a compact metric space and let $C(x)$ be the set of all complex valued, continuous, bounded functions on X. Define $||f|| = \sup_{x \in X} |f(x)|$. Then

(A) $C(x)$ is closed
(B) $C(x)$ is compact
(C) $C(x)$ is complete metric space
(D) None of the above

67. Let $C(x)$ be the set of all complex valued, continuous, bounded functions on X. Define $||f|| = \sup_{x \in X} |f(x)|$. A sequence of functions $\{f_n\}$ converges to f with respect to a metric of $C(x)$ if and only if

(A) $\{f_n\} \to f$ on X
(B) $\{f_n\} \to f$ on X and f is continuous
(C) $\{f_n\} \to f$ on X and f is bounded
(D) $\{f_n\} \to f$ uniformly on X

68. Let $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$ and $\{f_n\} \to f$ uniformly on $[a, b]$. Then $f \in \mathcal{R}(\alpha)$ if

(A) If f is continuous on $[a,b]$
(B) If f is differentiable on $[a,b]$
(C) If f is monotonic on $[a,b]$
(D) If α is monotonically increasing on $[a,b]$

69. Let $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$ and let $\sum f_n(x) = f(x); x \in [a,b]$. Then $\int_a^b f(x)d\alpha = \sum_n^\infty \int_a^b f_n(x)d\alpha$ if
(A) fn converges
(B) fn converges uniformly
(C) $\sum f_n$ converges to a bounded function
(D) None of the above

70. Let α is monotonically increasing on $[a,b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$; $n = 1,2,3, ...$ and $\{f_n\} \to f$ uniformly on $[a,b]$. Then

A) $f \in \mathcal{R}(\alpha)$
B) f is bounded
C) f is continuous
D) f is monotonically increasing

71. Which one of the following statements are correct
A. Uniform convergence of $\{f_n\}$ implies uniform convergence of $\{f_n'\}$
$B.$ $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$; $n = 1,2,3, ...$ and $\{f_n\} \to f$ uniformly on [a,b] implies $f \in \mathcal{R}(\alpha)$
C. Pointwise converges of a sequence implies uniform convergence of that sequence
D.None of the above

72. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on E. Then
$A.$ $\{f_n + g_n\}$ converge uniformly on E
$B.$ $\{f_n\ g_n\}$ converge uniformly on E
C. Both (A) and (B)
D. Neither (A) or (B)

73. If $\{f_n\}$ and $\{g_n\}$ converge uniformly on E. Then $\{f_n\ g_n\}$ converge uniformly on E if
$A.$ $\{f_n\}$ and $\{g_n\}$ are continuous
$B.$ $\{f_n\}$ and $\{g_n\}$ are differentiable
$C.$ $\{f_n\}$ and $\{g_n\}$ are bounded functions
$D.$ $\{f_n\}$ and $\{g_n\}$ are monotonic functions

74. Which one of the following sequences are uniformly convergent
$A.$ $f_n(x)=\frac{1}{nx+1}$; $x \in [0,1]$
$B.$ $f_n(x)=\frac{x}{nx^2+1}$; $x \in [0,1]$
C. Both (A) and (B)
D. Neither (A) nor (B)

75. Which of the following statements are false
A. There exists real continuous function on the real line which is differentiable
B. There exists no real continuous function on the real line which is nowhere differentiable
C. There exists real continuous function on the real line which is integrable
D. All of these

76. A sequence of functions $\{f_n\}$ is said to be pointwise bounded on E if

A. The sequence $\{f_n(x)\}$ is bounded for some $x \in E$
B. The sequence $\{f_n(x)\}$ is convergent for every $x \in E$
C. The sequence $\{f_n(x)\}$ is bounded for every $x \in E$
D. The sequence $\{f_n(x)\}$ is convergent for some $x \in E$

77. Let $\{f_n\}$ be a pointwise bounded sequence of complex functions on a set E. Then $\{f_n\}$ has a convergent subseuence in E if

A. E is compact
B. E is countable
C. E is closed
D. E is bounded

78. A family F of complex functions f defined on a set E in a metric space X is said to be equicontinuous on E if

A. for every $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - g(y)| < \varepsilon$ whenever $d(x, y) < \delta,\ x, y \in E$
B. Each function in F is continuous on E
C. Each function in F is uniformly continuous on E
D. for every $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - f(y)| < \varepsilon$ whenever $d(x, y) < \delta,\ x, y \in E\ and\ f \in F$

79. Let $\{f_n\}$ be a sequence of functions defined on a set E. If there exists a number M such that$|f_n(x)| < M\ for\ all\ x \in E$, then we say that $\{f_n\}$ is

A. Uniformly bounded on E
B. Uniformly continuous on E
C. Convergent on E
D. Uniformly convergent on E

80. The Stone – Weierstrass Theorem states that

A. If f is a complex function on $[a, b]$, then $\exists$ a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a, b]$.
B. If f is a continuous complex function on $[a, b]$, then $\exists$ a sequence of polynomials P_n such that $P_n(x)$ converges to $f(x)$ pointwise on $[a, b]$.
C. If f is a continuous complex function on $[a, b]$, then $\exists$ a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a, b]$.
D. If f is a complex function on $[a, b]$, then $\exists$ a sequence of polynomials P_n such that $P_n(x)$ converges to $f(x)$ pointwise on $[a, b]$.

81. f is said to be expanded in a power series about the point $x = a$ if

A. $f(x) = \sum_{n=0}^{\infty} c_n(x + a)^n$ converges for $|x - a| < R$, for some $R > 0$.
B. $f(x) = \sum_{n=0}^{\infty} ax^n$ converges for all $x \in (-R, R)$, for some $R > 0$.
C. $f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n$ converges for all $x \in (-R, R)$, for some $R > 0$
D. $f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n$ converges for $|x - a| < R$, for some $R > 0$

82. Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$ for $|x| < R$. Then

A. f is continuous but not differentiable in $(-R, R)$
B. f is not continuous and differentiable in $(-R, R)$

C. f is differentiable and $f'(x) = \sum_{n=1}^{\infty} nc_n x^{n-1}$ for $|x| < R$
D. $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R, R]$

83. Suppose $\sum_{n=0}^{\infty} c_n$ converges and put $f(x) = \sum_{n=0}^{\infty} c_n x^n$ where $-1 < x < 1$. Then what will be the limit $\lim_{x\to 1} f(x)$?
    A. $\sum_{n=0}^{\infty} c_n$
    B. -1
    C. 1
    D. Does not exist

84. Suppose the series $\sum a_n x^n$ and $\sum b_n x^n$ converge in the segment $S = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum a_n x^n = \sum b_n x^n$. Then what is the condition for satisfying $\sum a_n x^n = \sum b_n x^n$ for all $x \in S$?
    A. E is bounded
    B. E has a limit point in S
    C. E is a finite set
    D. $E = \emptyset$

85. The series $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ converges
    A. For every complex number z
    B. Only for those complex number z whose real part is greater than or equal to 0
    C. Only at $z = 0$
    D. Only for those complex number z whose imaginary part is greater than or equal to 0

86. If we define $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$, then which of the following is true?
    A. $\mathrm{E(Z+W)} = \mathrm{E(Z)+E(W)}$
    B. $\mathrm{E(Z+W)} \leq \mathrm{E(Z)+E(W)}$
    C. $\mathrm{E(Z+W)} = \mathrm{E(Z).E(W)}$
    D. $\mathrm{E(Z+W)} \geq \mathrm{E(Z)+E(W)}$

87. What is the product $E(z).E(-z)$ where $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$
    A. 0
    B. 1
    C. ∞
    D. -1
    E.

88. If we define $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$, which of the following is true?
    A. $E(z) = 0$ for some z
    B. $E(x) < 0$ for all real x
    C. $E(x) < 0$ if $x > 0$
    D. $E(z) \neq 0$ for all z

89. Which of the following is not a property of $E(z)$ where $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$
    A. $E(x) \to 0 as\ x \to -\infty$ along the real axis
    B. E is strictly decreasing on the whole real axis
    C. E is strictly increasing on the whole real axis

D. $E(x) \to +\infty \; as \; x \to +\infty$ along the real axis

90. For every n, what is the limit $\lim_{x \to +\infty} x^n e^{-x}$?
   A. $+\infty$
   B. $-\infty$
   C. 0
   D. 1

91. What is mean by the algebraic completeness of the Complex Field?
   A. Every non constant polynomial with complex coefficients has a complex root.
   B. Every non constant polynomial with complex coefficients has exactly one complex root.
   C. Every polynomial with complex coefficients has atmost one complex root.
   D. Every non constant polynomial has no any complex root.

92. If $C(x) = \frac{1}{2}[E(ix) + E(-ix)]$, then which of the following is true?
   A. $C(0) = 0$
   B. $C(x) \neq 0 \; \forall x$
   C. There exist positive integers x such that $C(x) = 0$
   D. $C'(x) = S(x)$ where $S(x) = \frac{1}{2i}[E(ix) - E(-ix)]$

93. The function E is periodic with period
   A. 2π
   B. $2\pi i$
   C. πi
   D. π

94. The functions C and S are periodic with period
   A. $2\pi i$
   B. π
   C. πi
   D. 2π

95. If there is a unique t in $[0, 2\pi)$ such that $E(it) = z$, then
   A. $|z| < 1$
   B. $|z| > 1$
   C. $|z| = 1$
   D. $z = 0$

96. Every power series is
   A. Convergent
   B. Divergent
   C. Uniformly Convergent
   D. Nowhere Convergent

97. If $S(x) = \frac{1}{2!}[E(ix) - E(-ix)]$,then
   A. $\overline{S(x)} = S(x)$
   B. $S(0) = 0$

C. $S'(x) = C(x)$, where $C(x) = \frac{1}{2}[E(ix) - E(-ix)]$

D. $S\left(\frac{\pi}{2}\right) = 1$

98. Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$ and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $|x| < R$. Then $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\varepsilon, R-\varepsilon]$ no matter which $\varepsilon > 0$ is chosen. The function f is continuous and differentiable in

A. $(-R+\varepsilon, R-\varepsilon)$

B. $[-R+\varepsilon, R-\varepsilon]$

C. $[-R, R]$

D. $(-R, R)$

99. Let $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}, 0 \le x \le 1, n = 1,2,3 \ldots..$ Then which statement is true.

A. $\lim_{n\to\infty} f_n(x) = 0$

B. $\{f_n\}$ is uniformly bounded on $[0,1]$

C. $f_n\left(\frac{1}{n}\right) = 1$

D. Subsequence converges uniformly on $[0,1]$

100. When we say that f is expanded on a power series about the point $x = a$

A. If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ converges $\forall\, x \in (-R, R)$

B. If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ converges for $|x-a| < R$

C. If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ converges for $|x-a| > R$

D. If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ diverges $\forall\, x \in (-R, R)$