PG SEMESTER I
ME010102:Linear Algebra
Multiple Choice Questions

1. Which of the following is not a vector space?
   (a) R over the field R
   (b) N over the field R
   (c) C over the field Q
   (d) The set of all functions from R^2 to R over the field R

2. Let V be the real vector space of all functions $f : R \to R$. If $V_1 = \{f \in V | f(0) = f(1)\}$ and $V_2 = \{f \in V | f(-x) = -f(x)\}$
   (a) Neither V_1 nor V_2 are subspaces of V
   (b) V_1 is a subspace of V but V_2 is not a subspace of V
   (c) V_1 is not a subspace of V but V_2 is a subspace of V
   (d) both V_1 and V_2 are subspaces of V

3. If the dimensions of subspaces V_1 and V_2 of a vector space V are 8 and 10 respectively and $dim(V_1 + V_2) = 4$, then $dim(V_1 \cap V_2)$ is
   (a) 10
   (b) 14
   (c) 22
   (d) 8

4. The dimension of the subspace $W = \{(x, y, z)|x - 2y = 0\}$ of R^3 is
   (a) 1
   (b) 2
   (c) 3
   (d) 0

5. Which of the following are linearly independent subsets of R^3
   (a) $(1, 4, 0), (1, 2, 1), (-5, 0, 6)$
   (b) $(1, 3, 0), (2, 0, 1), (3, 3, 1)$
   (c) $(1, 1, 3), (-1, 0, 2), (-4, 0, 8)$

(d) $(1,1,0),(1,-3,-4),(-1,1,2)$

6. Which of the following is a basis of the vector space R^3

   (a) $(1,0,0),(0,1,1)$
   (b) $(1,2,0),(0,-1,3),(-1,-4,6)$
   (c) $(1,1,0),(0,0,1),(1,1,1)$
   (d) $(1,0,0),(0,1,1),(1,0,1),(1,1,0)$

7. Which of the following is not a property of a vector space V over the field F where $\alpha,\beta \in V$ and $a,b \in F$.

   (a) $(a+b)\alpha = a\alpha \ + b\alpha$
   (b) $a(b\alpha) = (ab)\alpha$
   (c) $\alpha\beta = \beta\alpha$
   (d) $\alpha + \beta \ \in V$

8. Determine the scalars a,b,c so that the vector $(1,1,1)$ is a linear combination of $(1,2,3),\ (1,0,1),\ (1,1,0)$

   (a) $a = 1/4,\ b = 1/4,\ c = 1/2$
   (b) $a = 1/2,\ b = 1/4,\ c = 1/4$
   (c) $a = 1/4,\ b = 1/2,\ c = 1/4$
   (d) $a = 1/4,\ b = 1/4,\ c = 1/4$

9. Which of the following subsets are subspace of the vector space of all n x n matrices over R

   (a) Set of all matrices whose diagonal elements are all 0
   (b) Set of all matrices whose diagonal elements are all 1
   (c) Set of all matrices whose determinant is 1
   (d) Set of all matrices whose trace is 0

10. Let V be a vector space over a field F. Let V_1 and V_2 be subspaces of V. Then

    (a) $V_1 \ \cup \ V_2$ is a subspace of V
    (b) $V_1 \ \cap \ V_2$ is a subspace of V
    (c) $V_1 \ + \ V_2$ is not a subspace of V
    (d) $V_1 \setminus V_2$ is a subspace of V

11. Which of the following vectors not span the vector space R^3

    (a) $(1,2,3),\ (2,5,6),\ (0,0,1)$

(b) $(1,1,0)$, $(0,1,1)$, $(1,0,1)$ $(1,1,1)$

(c) $(1,5,2)$, $(2,5,1)$, $(1,3,1)$ $(4,1,2)$

(d) $(1,0,0)$, $(0,1,0)$, $(4,5,0)$

12. Which of the following statement is false

(a) Any set that contains a linearly dependent set is linearly dependent

(b) Any set which contains the 0 vector is linearly dependent

(c) Any set that contains a linearly independent set is linearly independent

(d) A set S of vectors is linearly dependent if and only if each finite proper subset of S is linearly dependent.

13. Let V be a finite dimensional vector space and let $n = dim(V)$. Which of the following statements are false

(a) Any subset of V which contains more than n vectors is linearly dependent

(b) No subset of V which contains less than n vectors can span V

(c) Any set which spans V is linearly independent

(d) Any two bases of V have the same (finite) number of elements

14. Which of the statement is false

(a) If two vectors are linearly dependent, then one of them is a scalar multiple of the other

(b) Linearly independent subset of a finite dimensional vector space is finite

(c) Proper subset of a basis is a spanning set

(d) The vector space R over Q is infinite dimensional

15. Let V be the vector space of all mxn matrices over the field F. Then $dim\ V =$

(a) $m + n$

(b) mn

(c) $m - n$

(d) n^2

16. Let A and B be mxn matrices over the field F. Let A and B are row-equivalent. Then

(a) A and B have the same row space

(b) A and B have different row space

(c) $B = PA$, where P is an invertible mxm matrix

(d) The homogeneous systems $AX = 0$ and $BX = 0$ have the same solutions

17. Let V be the vector space of all 2x2 matrices over R. consider the subspaces W_1 and W_2 of the form $W_1 = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$ and $W_2 = \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix}$, where x, y are scalars. If $m = dim(W_1 \cap W_2)$ and $n = dim(W_1 + W_2)$, then

(a) $m = 1, n = 3$

(b) $m = 3, n = 1$

(c) $m = 2, n = 3$

(d) $m = 3, n = 2$

18. The dimension of subspace $W = \{(x, y, z)|x + y + z = 0\}$ of R^3 is

(a) 0

(b) 1

(c) 2

(d) 3

19. The number of subspaces of R^2

(a) 1

(b) 2

(c) Finite

(d) Infinite

20. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following is true

(a) The zero vector of V is an element of B

(b) B has a proper subset that spans S

(c) B is a proper subset of a linearly independent subset of V

(d) There is a basis for V that is disjoint from B

21. Which of the following vectors are linearly independent

(a) $\{1, x, 2x + 7\}$

(b) $\{7, x^2, x^3\}$

(c) $\left\{ \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}, \begin{bmatrix} 7 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 9 & 6 \\ 1 & 14 \end{bmatrix} \right\}$

(d) (1,1,1), (1,2,3), (2,1,0)

22. If V_1 and V_2 are 3-dimensional subspaces of a 4-dimensional vector space V, then the smallest possible dimension of $V_1 \cap V_2$ is

(a) 0
(b) 1
(c) 2
(d) 3

23. Let V be the vector space of all polynomials of degree ≤ 5 over R. Then dimension of V is

(a) 5
(b) 6
(c) 7
(d) 8

24. Consider the vector space R^2 over R and let $S = (4, 0)$. Then span(S) is

(a) S
(b) $\{(x, 0)|x \in R\}$
(c) $\{(0, y)|x \in R\}$
(d) R^2

25. Which of the following is an infinite dimensional vector space

(a) R^2 over R
(b) C over R
(c) R over Q
(d) R over R

26. What is a linear operator?

(a) A linear operation from a set to itself.
(b) A linear operaton from a vector space to itself.
(c) A linear transformation from a vector space to itself.
(d) A linear transformation from a vectoe space to another vector space.

27. Which of the following is correct for a vector space V and a linear transformation T.

(a) *Rank(V) + Nullity (V) = dim (T)*
(b) *Rank(T) + Nullity (T) = dim (V)*
(c) *Rank(V) + Nullity (T) = dim (T)*
(d) *Rank(T) + Nullity (V) = dim (T)*

28. Let V be an m-dimensional vector space over the field and W be an n-dimensional vector space over a field over a field F. Then what is the dimension of L(V,W).

(a) mn

(b) m+n

(c) m-n

(d) $\frac{m}{n}$

29. Inverse of a linear transformation is a linear transformation.

(a) Never

(b) Some times

(c) Always

(d) Depends on the matrix related to the linear transformation.

30. If U and T are linear transformations, then

(a) $(UT)^{-1} = U^{-1}T^{-1}$

(b) $(UT)^{-1} = (T^{-1}U^{-1})^{-1}$

(c) $(UT)^{-1} = (U^{-1}T^{-1})^{-1}$

(d) $(UT)^{-1} = T^{-1}U^{-1}$

31. Every n-dimensional vector space over the field F is isomorphis to the space

(a) nF

(b) F^n

(c) Fn

(d) n^F

32. Two n × n matrices are similar, if

(a) They have same rank.

(b) they have same determinant

(c) If there exixt an n ×n matrix P such that $B = P^{-1}AP$

(d) None of the above.

33. Suppose V is a vector space with dimension n. The a subspace of V with a dimension $n - 1$ is called.

(a) Hyperspace

(b) Hyperplane

(c) Subspace

(d) Hyper subspace.

34. Annihilator of a set in a vector space is

    (a) Is a set of linear transformations
    (b) Is a set of linear functional
    (c) A single linear transformation.
    (d) A single linear functional.

35. The sum of dimensions of a subspace and its annihilator is.

    (a) Dimension of the full space
    (b) 0
    (c) < 0
    (d) > 0

36. A linear functional is

    (a) A function from a vector space to itself.
    (b) A linear transformation from a vector space to the field.
    (c) A linear transformation from a vector space to itself.
    (d) A function from a vector space to field.

37. Which of the following statement is true about trace(A)

    (a) It is the sum of diogonal elements of A.
    (b) It is a linear functional.
    (c) Both A and B
    (d) Neither A nor B

38. Rank of a linear functional is

    (a) 0
    (b) 1
    (c) Can take any value
    (d) None.

39. Find the wrong one from the given statements.

    (a) If A is diagonalizable and invertible, then A^{-1} is diagonalizable.
    (b) If A is diagonalizable, then A^T is diagonalizable.

(c) If every eigenvalue of a matrix A has algebraic multiplicity 1, then A is diagonalizable

(d) An n × n matrix with fewer than n distinct eigenvalues is not diagonalizable.

40. Let T : V → V be a linear operator and T (x) = λx for some scalar λ. Then x is called

(a) an eigenvector of T .

(b) an eigenvalue of T .

(c) an eigenspace of T

(d) None of these.

41. If A is an m × n matrix, then the null space of A

(a) a subspace of R^n

(b) a subspace of R^m

(c) a subspace of R^{mn}

(d) a subspace of $R^{min(m,n)}$

42. The rank of a matrix A is the

(a) dimension of the row space of A.

(b) dimension of the column space of A.

(c) both A and B

(d) dimension of the null space of A.

43. Let T : $R^5 \to R^3$ be the linear transformation defined by $T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2, x_2 + x_3 + x_4, x_4 + x_5)$. Find the nullity of the standard matrix for T.

(a) 5

(b) 3

(c) 2

(d) 1

44. For a matrix A, the row space of A^T is same as

(a) row space of A

(b) column space of A

(c) column space of A^T

(d) null space of A

45. Which of the following is true?

(a) Every linearly independent set of five vectors in R^5 is a basis for R^5

(b) Every set of five vectors that spans R^5 is a basis for R^5

(c) Every set of vectors that spans R^5 cotains a basis for R^5

(d) All are true

46. If A and B are square matrices of the same order, then tr(AB) =

    (a) tr(A + B)

    (b) tr(A)tr(B)

    (c) tr(BA)

    (d) tr(A) + tr(B)

47. Which of the following is a subspace of R^3?

    (a) All vectors of the form (a, 0, 0)

    (b) All vectors of the form (a, 1, 1)

    (c) All vectors of the form (a, b, c) where b = a + c + 1

    (d) None of these

48. Let f = cos(2x), g = sin(2x). Which of the following lie in the space spanned by f and g?

    (a) $3 + x^2$

    (b) sin(x)

    (c) Both A and B

    (d) Neither A nor B

49. What is the maximum possible rank of an m × n matrix A that is not square?

    (a) rank(A) $\geq$ min(m, n)

    (b) rank(A) $\leq$ min(m, n)

    (c) rank(A) = min(m, n)

    (d) rank(A) = max(m, n)

50. If 0 is an eigenvalue of a matrix A, then the set of columns of A is

    (a) linearly independent or linearly dependent.

    (b) linearly dependent always.

    (c) linearly independent always.

    (d) Cannot be determined

51. Which of the following function on the set of 3×3 matrices is 3-linear?

(a) $D(A) = A_{11} + A_{22} + A_{33}$
(b) $D(A) = 0$
(c) $D(A) = A_{11}A_{22}A_{33}$
(d) $D(A) = 1$

52. If A is an n×n matrix with real entries and rows of A are linearly independent, then

(a) $detA = 0$
(b) $detA \neq 0$
(c) $detA < 0$
(d) $detA > 0$

53. Let D be an alternating 2-linear function on the set of 2×2 matrices over a field F. Then which of the following statement is true for any two rows α and β of the matrix?

(a) $D(\alpha, \beta) = D(\beta, \alpha)$
(b) $D(\alpha, \beta) = -D(\beta, \alpha)$
(c) $D(\alpha, \beta) = 0$
(d) $D(\alpha, \beta) = D(\alpha, \alpha)$

54. The determinant function on the set of n×n matrices over the field F is not

(a) one-one
(b) onto
(c) n-linear
(d) alternating

55. If A is an nn matrix over a field R with $detA = 30$, then the determinant of the matrix obtained by interchanging two columns of A is

(a) -30
(b) 1/30
(c) 3.30
(d) 0

56. Which of the following is not always true?

(a) If A is an n×n matrix over the field F, $det(A^t) = detA$.
(b) If A and B are n×n matrices over the field F, $det(A + B) = detA + detB$
(c) If A and B are n×n matrices over the field F, $det(AB) = (detA)(detB)$

(d) If A is an invertible n×n matrix over the field F, $det(A^{-1}) = (detA)^{-1}$.

57. If A is an n×n matrix over a field F with $AA^t = I$, then

   (a) $A = I$
   (b) A is non-singular
   (c) A is singular
   (d) $detA = 1$

58. If A and B are n×n matrices over the field F, A non-singular and AB=0,then B is

   (a) Zero matrix
   (b) Identity matrix
   (c) Non-singular matrix
   (d) A^{-1}

59. Let A be an n×n skew-symmetric matrix with complex entries and n is odd, then $detA$ is

   (a) 1
   (b) -1
   (c) 0
   (d) $(-1)^n$

60. The determinant of a triangular matrix is

   (a) the product of its diagonal entries
   (b) always zero
   (c) the sum of its diagonal entries
   (d) always positive

61. If A and B are similar matrices over a field F, then

   (a) $detB = detA^{-1}$
   (b) $detB = (detA)^{-1}$
   (c) $detB = detA$
   (d) $detB = 0$

62. Which of the following is not a property of determinant?

   (a) Determinant function is linear.
   (b) If two rows of a matrix are equal, then its determinant is zero.

(c) The value of non-zero determinant of a matrix changes if any two rows of the matrix are interchanged.

(d) The determinant remains unchanged if a matrix is obtained by adding a multiple of one row of the matrix to another row of the same matrix.

63. If A is a 2×2 matrix over R with $det(A+I) = 1 + detA$, then which of the following is always correct?

(a) $detA = 0$

(b) $A = 0$

(c) $trace(A) = 0$

(d) $detA = 1$

64. If $A^2 - A + I = 0$, then $A^{-1} =$?

(a) A^2

(b) $A + I$

(c) $A - I$

(d) $I - A$

65. If A and B are two n×n matrix over R with $c \in R$, then which of the following is always true?

(a) $det(cAB) = c^n det(A)det(B)$

(b) $det(cA - B) = cdet(A) - det(B)$

(c) $det(cA - B) = cdet(A) + det(B)$

(d) $det(cAB) = cdet(A)det(B)$

66. Let σ and τ be the permutations of degree 4 defined by $\sigma 1$=2,$\sigma 2$ =3,$\sigma 3$ =4,$\sigma 4$ =1, $\tau 1$=3,$\tau 2$=1,$\tau 3$=2,$\tau 4$=4. Then,

(a) both σ and τ are odd permutations

(b) both σ and τ are even permutations

(c) σ is odd and τ is even permutation

(d) σ is even and τ is odd permutation

67. If A is an 2×2 matrix over R such that A(adj A)=$\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$, then $detA =$?

(a) 4

(b) 8

(c) 2

(d) 16

68. If A is an n×n matrix over a field F and $c \in F$, then which of the following is false?

(a) $det(A^t) = det(A)$

(b) $det(cA) = c^n det(A)$

(c) $adj(A^t) = adj(A)$

(d) $adj(A^t)A^t = det(A)I$

69. If A= $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is a matrix over the field F, then

(a) $detA = 1$

(b) $detA = abc$

(c) $detA = -abc$

(d) $detA = 0$

70. If A and B are invertible n×n matrices over a field F, then which of the following is always true?

(a) A+B is invertible

(b) $A^{-1} + B^{-1}$ is invertible

(c) AB is invertible

(d) A-B is invertible

71. An n×n matrix A over the polynomial ring F[x] is invertible over F[x] if and only if

(a) $detA = 0$

(b) detA is a non-zero polynomial in F[x] .

(c) detA is a non-zero polynomial of degree less than or equal to 1 in $F[x]$.

(d) detA is a non-zero scalar polynomial in $F[x]$.

72. If A is an n×n matrix over a field F with a left inverse, then

(a) right inverse of A may or may not exists.

(b) right inverse of A exists but A is not invertible.

(c) right inverse of A exists but both inverses may differ.

(d) right inverse of A exists and A is invertible.

73. A is an n×n matrix over a field F, then the classical adjoint of A, adj A is defined by

(a) $(adjA)_{ij} = (-1)^{i+j} detA(i|j)$

(b) $(adjA)_{ij} = (-1)^{i+j} detA(j|i)$

(c) $(adjA)_{ij} = (-1)^{i+j} detA(i|i)$

(d) $(adjA)_{ij} = (-1)^{i+j} detA(j|j)$

74. Let S denotes the set of all primes p such that the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -2 & 0 & 2 \end{bmatrix}$ is invertible when considered as a matrix in Z_p. Then

(a) S is empty

(b) S contains all prime numbers greater than 10

(c) S contains all odd prime numbers

(d) S contains all prime numbers less than 10

75. The determinant of the matrix $\begin{bmatrix} 2 & 5 & 2 & 5 \\ 2 & 2 & 0 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 8 & 6 \end{bmatrix}$ is

(a) 36

(b) -24

(c) 12

(d) 0

76. The determinant function on the set of n×n matrices over a field F is not

(a) linear

(b) n-linear

(c) alternating

(d) unique

77. If A is an n×n matrix over a field F, then the matrix A is invertible if and only if

(a) is a non-zero matrix

(b) A is symmetric

(c) A is non-singular

(d) A=I

78. If A is a square matrix of order 3 and $detA = 5$, then $det(3A)$ is

(a) 135

(b) 405

(c) 125

(d) 15

79. If A is any n×n matrix over a field F, then AA^t is

(a) Identity matrix

(b) Symmetric matrix

(c) Skew-symmetric matrix

(d) Zero matrix

80. If D is an alternating function on the set of n×n matrices over a field F and A' is the matrix obtained by interchanging any two rows of A, then $D(A')$ is

(a) $D(A)$

(b) 0

(c) $-D(A)$

(d) 1

81. The characteristic polynomial of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

(a) $x^2 + 1$

(b) $x^2 - x + 1$

(c) $1 - x^2$

(d) $x^2 + x + 1$

82. Which of the following is the characteristic polynomial of an $n \times n$ matrix A.

(a) $f(x) = det(xI - A)$

(b) $f(x) = det(xI - A^2)$

(c) $f(x) = det(xI + A)$

(d) $f(x) = det(I - xA)$

83. The characteristic values of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ are

(a) $1, 2$

(b) $1, -2$

(c) $-1, 2$

(d) $-1, -2$

84. The characteristic values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ are

(a) $-2, 3, 2$

(b) $0, 3, 1$

(c) $-1, 0, 2$

(d) $1, 3, 2$

85. If c is the characteristic value of an $n \times n$ matrix A, then

(a) $det(A) = c$

(b) $det(cI - A) = 0$

(c) $det(A + I) = c$

(d) $det(A + cI) = 0$

86. Let T be a linear operator on the finite dimensional vector space V. T is diagonalizable if there is a basis for V in which each vector is a .... of T.

(a) Characteristic Value

(b) Characteristic Vector

(c) Singular Value

(d) Singular Vector

87. The characteristic polynomial of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ is

(a) $f(x) = x(x - 1)$

(b) $f(x) = (x + 1)(x - 1)$

(c) $f(x) = x(x + 1)$

(d) $f(x) = (x - 1)(x - 2)$

88. Let T be a linear operator on the finite dimensional vector space V over the field F. The unique monic generator of the ideal of polynomials over F which annihilate T is called

(a) Minimal Polynomial.

(b) Symmetric Polynomial.

(c) Singular Polynomial.

(d) Characteristic Polynomial.

89. Let T be a linear operator on the finite dimensional vector space V over the field F. If f is the characteristic polynomial for T, then $f(T)$ is

(a) $dimV$

(b) 1

(c) -1

(d) 0

90. Choose the correct statement

(a) Characteristic polynomial divides the minimal polynomial.

(b) Characteristic polynomial and minimal polynomial has the same degree.

(c) Minimal polynomial divides the characteristic polynomial.

(d) The sum of minimal and characteristic polynomial is the zero polynomial.

91. Let T be a linear operator on $\mathbb{R}^2$ which is represented in the standard ordered basis by a matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then

(a) the only subspace of $\mathbb{R}^2$ which is invariant under T is $\mathbb{R}^2$.

(b) the only subspace of $\mathbb{R}^2$ which is invariant under T is the zero subspace.

(c) the only subspaces of $\mathbb{R}^2$ which are invariant under T are $\mathbb{R}^2$ and the zero subspace.

(d) there are three subspaces of $\mathbb{R}^2$ which are invariant under T.

92. Let T be a linear operator on a vector space V. A subspace W is invariant under T if

(a) $T(W) = \phi$

(b) $T(W) = W$

(c) $T(W) \subset W$

(d) $T(W) \cup W = \{0\}$

93. Suppose that E is a projection on a vector space V. Let R be the range of E and let N be the null space of E. Then

(a) $V = R \cup N$

(b) $V = R \cap N$

(c) $R \oplus N = \phi$

(d) $V = R \oplus N$

94. A linear operator E on a vector space V is called a projection if

(a) $E^2 = -E$

(b) $E^2 = E$

(c) $E^2 = I$

(d) $E^2 = -I$

95. Let $P : \mathbb{R}^2 \to \mathbb{R}^2$ be defined as $P(x, y) = (x, 0)$. Then

(a) P is a projection

(b) P is not a projection

(c) $P^2 = -P$

(d) $P^2 = I$

96. Let V be a finite dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \dots \alpha_n\}$ be any basis for V. If W_i is the one dimensional subspace spanned by α_i, then

(a) $V = W_1 \oplus W_2 \oplus \cdots \oplus W_n$

(b) $W_1 \oplus W_2 \oplus \cdots \oplus W_n = \phi$

(c) $V = W_1 \cup W_2 \cup \cdots \cup W_n$

(d) $V = W_1 \cap W_2 \cap \cdots \cap W_n$

97. Let n be a positive integer and F a subfield of the complex numbers and let V be the space of all $n \times n$ matrices over F. Let W_1 be the space of all symmetric matrices and W_2 the space of all skew symmetric matrices. Then

(a) $V = W_1 \cap W_2$

(b) $V = W_1 \oplus W_2$

(c) $V = W_1 \cup W_2$

(d) $W_1 \oplus W_2 = \phi$

98. Suppose that E is a projection. Let R be the range of E and let N be the null space of E. The unique expression for $\alpha \in V$ as a sum of vectors in R and N is

(a) $\alpha = E\alpha + (\alpha - E\alpha)$

(b) $\alpha = E\alpha + (\alpha + E\alpha)$

(c) $\alpha = E\alpha + (E\alpha - \alpha)$

(d) $\alpha = E\alpha + (-\alpha - E\alpha)$

99. Let T be any linear operator on a finite dimensional space V. Let $c_1, \dots, c_k$ be the distinct characteristic values of T, and let let W_i be the space of characteristic vectors associated with the characteristic value c_i. Suppose that T is diagonlizable then

(a) $V = W_1 \cap \cdots \cap W_k$

(b) $V = W_1 \cup \cdots \cup W_k$

(c) $V = W_1 \oplus \cdots \oplus W_k$

(d) $W_1 \cup \cdots \cup W_k = \phi$

100. Consider the following statements
1. Similar matrices have the same characteristic polynomial.
2. Characteristic values of a diagonal matrix are diagonal entries.
Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

101. Find the matrix with two distinct characteristic values

(a) $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} -1 & 0 \\ 2 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2 & -2 \\ 0 & 2 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

102. The characteristic polynomial of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is

(a) $f(x) = x^2 - 2x + 1$

(b) $f(x) = x^2 + 2x + 1$

(c) $f(x) = x^2 + 2x - 1$

(d) $f(x) = x^2 - 2x - 1$

103. Let A be the 2×2 matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Choose the correct statement

(a) 1 and -1 are the two characteristic values of A.

(b) A has two characteristic values in $\mathbb{R}$.

(c) A has no characteristic values in $\mathbb{C}$.

(d) A has two characteristic values in $\mathbb{C}$.

104. Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Then

(a) A has only real characteristic values

(b) A has no real characteristic values

(c) A is skew-symmetric

(d) Sum of the characteristic values of A is 8

105. Let $A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$.Find the sum of the eigen values

(a) 1

(b) -2

(c) -1

(d) 0