M.Sc. Mathematics

Sem 1 Abstract Algebra

Multiple Choice Questions

1. What is the order of the group $Z_3 \times Z_4$

A. 11 C. 12

B. 15 D. 16

2. What is the order of element (2,3) in the group $Z_6 \times Z_{15}$

A. 15 C. 19

B. 16 D. 12

3. Are the groups and $Z_2 \times Z_{12}$ and $Z_4 \times Z_6$ isomorphic

A. Yes B. No

4. Find the order of the factor group $Z_4 \times Z_2$/<(2,1)>

A. 6 C. 4

B. 8 D. 3

5. Order of the element (0,0) in the group $Z_2 \times Z_4$

A. 1 C. 3

B. 2 D. 4

6. Order of the element (3,10,9) in the group $Z_4 \times Z_{12} \times Z_{15}$

A. 50 C. 70

B. 60 D. 80

7. Order of the element 5+<4> in Z_{12}/<4>

A. 6 C. 8

B. 4 D. 10

8. Find the order of the Factor group Z_6/<3>

A. 2 C. 4

B. 3 D. 6

9. Find the order of the Factor group ($Z_{12} \times Z_{18}$)/<(4,3)>

A. 36 C. 30

B. 12 D. 18

10. Find the order of the element 26+<12> in the Factor group $Z_{60}/<12>$

A. 8 C. 6

B. 10 D. 12

11. Number of all proper nontrivial subgroups of $Z_2 \times Z_2 \times Z_2$

A. 3 C. 6

B. 5 D. 7

12. Find all proper non trivial subgroups of $Z_2 \times Z_2$

A. {(0,0),(1,0)} {(0,0),(0,1)} {(0,0),(1,1)}

B. {(0,0),(1,0)}

C. {(0,0),(1,0)} {(0,0),(0,1)}

D. {(0,0),(1,0)} {(0,0),(1,1)}

13. How many abelian groups (up to isomorphism) are there of order 24

A. 1 C. 5

B. 3 D. 7

14. Find all abelian groups of order 8 up to isomorphism

A. Z_8 C. $Z_4 \times Z_2$

B. $Z_2 \times Z_4$ D. $Z_2 \times Z_2 \times Z_2$

15. Find the order of the torsion subgroup of $Z_4 \times \mathrm{Z} \times Z_3$

A. 4 C. 12

B. 3 D. 16

16. Find the order of the torsion subgroup of $Z_{12} \times \mathrm{Z} \times Z_{12}$

A. 12 C. 24

B. 144 D. 132

17. How many subgroups of $Z_5 \times Z_6$ is isomorphic to $Z_5 \times Z_6$

A. 1 C. 3

B. 2 D. 4

18. Find the order of the factor group $(Z_{11} \times Z_{15})/<(1,1)>$

A. 5 C. 1

B. 4 D. 3

19. Find the torsion subgroup of the multiplicative group R^* of the non zero real numbers

A. {1,0} C. {-1,1}

B. {0,1} D. {-1,0}

20. Find the maximum possible order for some element of $Z_4 \times Z_{18} \times Z_{15}$

A. 180 C. 188

B. 186 D. 184

21. The Klein 4- group is isomorphic to which group

A. $Z_2 \times Z_4$ C. $Z_2 \times Z_6$

B. $Z_2 \times Z_2$ D. $Z_4 \times Z_2$

22. Find all abelian groups of order 16 up to isomorphism

A. $Z_2 \times Z_8$ C. $Z_4 \times Z_4$

B. $Z_2 \times Z_6$ D. $Z_3 \times Z_6$

23. Find the order of the given factor group $(Z_3 \times Z_5)/(\{0\} \times Z_5)$

A. 5 C. 7

B. 2 D. 3

24. Find the order of the given factor group $(Z_2 \times Z_4)/<(1,1)>$

A. 8 C. 2

B. 4 D. 6

25. Find the order of the element (8,10) in of the direct product $Z_{12} \times Z_{18}$

A. 12 C. 6

B. 3 D. 9

26. $(\mathbb{Z}/6\mathbb{Z})/(\mathbb{Z}/2\mathbb{Z})$ is

A. $\mathbb{Z}_6/\langle 2 \rangle$ C. $\mathbb{Z}/2\mathbb{Z}$

B. $\mathbb{Z}_2$ D. All the above

27. If $\varphi: \mathbb{Z}_{12} \to \mathbb{Z}_3$ be a homomorphism such that $\varphi(1) = 2$. Then the Kernel of φ is

A. $0, 3, 6, 9$ C. $0, 2, 4, 6, 8, 10$

B. 0,6 D. None of the above

28. If $\varphi: \mathbb{Z}_{18} \to \mathbb{Z}_{12}$ be a homomorphism such that $\varphi(1) = 10$. Then the group $\varphi[\mathbb{Z}_{18}]$ is

A. $0, 3, 6, 9$ C. $0, 2, 4, 6, 8, 10$

B. 0,6 D. None of the above

29. In $\mathbb{Z}_{24}$, let $H = \langle 4 \rangle$ and $N = \langle 6 \rangle$. Then, the elements in HN are

A. $\{0,2,4,6,8,10,12,14,16,18,20,22\}$ C. $\{0,8,16\}$

B. $\{0,4,8,12,16,20\}$ D. None of the above

30. In $\mathbb{Z}_{48}$, let $H = \langle 4 \rangle$ and $N = \langle 6 \rangle$. Then, the elements in $H \cap N$ are

A. $\{0\}$ C. $\{0,6,12,18,24,30,36,42\}$

B. $\{0,12,24\}$ D. $\{0,12,24,36\}$

31. In the group $G = \mathbb{Z}_{36}$, let $H = \langle 9 \rangle$ and $K = \langle 18 \rangle$. Then, the number of cosets in $(G/K)/(H/K)$ are

A. 12
B. 6
C. 9
D. 3

32. The order of Sylow 3-subgroup of a group of order 12 is

A. 3
B. 4
C. 12
D. None of the above

33. The number of Sylow 2 subgroups of a group of order 24 is

A. 1 or 3
B. 2 or 6
C. All of the above
D. None of the above

34. Which among the following is a p-group

A. $\mathbb{Z}_{36}$
B. $\mathbb{Z}_{16}$
C. $\mathbb{Z}_{15}$
D. $\mathbb{Z}_{100}$

35. Sylow 3-subgroups of S_6 are

A. $\langle(1,2,3)\rangle$
B. $\langle(1,2,4)\rangle$
C. $\langle(1,3,4)\rangle$
D. All the above

36. Which of the following groups is not solvable?

A. S_3
B. $\mathbb{Z}$
C. S_5
D. Z_9

37. Let G be a group of order 15. Then, the number of Sylow subgroup of G of order 3 is

A. 1
B. 0
C. 2
D. 3

38. A group of order ……….. is simple

A. 13
B. 48
C. 45
D. 36

39. Statement 1: Every group of order p^2 is abelian

Statement 2: Every group of order p^r for $r > 1$ is simple, where p is a prime

A. Both 1 and 2 are true
B. 1 is true, 2 is false
C. 1 is false, 2 is true
D. Both 1 and 2 are false

40. The center of a group G is nontrivial if G has order

A. 9
B. 8
C. 25
D. All the above

41. A Sylow-p-subgroup of a finite group G is

A. abelian

B. is unique if and only if it is normal in G

C. is cyclic

D. None of the above

42. Statement 1: Any two Sylow p-subgroups of a finite group are conjugates

Statement 2: Every p-subgroup of every finite group is a Sylow p-subgroup

A. 1 is true, 2 is false

B. 2 is true, 1 is false

C. Both are true

D. Both are false

43. The statement "If H is a subgroup of G and N is a normal subgroup of G, then $(HN)/N \simeq H/H \cap N$ is

A. Second isomorphism theorem

B. Third isomorphism theorem

C. Second Sylow theorem

D. Third Sylow theorem

44. If N is a normal subgroup of G and H is any subgroup of G, then

A. $H \vee N = HN \neq NH$

B. $H \vee N \neq HN = NH$

C. $H \vee N \neq HN \neq NH$

D. $H \vee N = HN = NH$

45. If G is a group of order p^n, X is a finite G-set, $X_G = \{x \in X \mid gx = xg \text{ for all } g \in G\}$ and $Gx = \{gx \mid g \in G\}$, then

A. $|X| \equiv |Gx| (\text{mod } p)$,

B. $|X| \equiv |X_G| (\text{mod } p)$,

C. $|X| = r + \sum_{i=1}^{r} |Gx_i|$, where r is the no. of orbits in X under G

D. $|X| = |X_G| + \sum_{i=1}^{r} |Gx_i|$

46. If G is a group of order 75, then

A. G has subgroups of order 3 and 5

B. G has subgroup of order 3 but no subgroup of order 5

C. G has subgroup of order 5 but no subgroup of order 3

D. G need not have subgroups of order 3 and 5

47. Let H be a p-subgroup of a finite group G.

Statement 1: $(N[H]: H) \neq (G: H) (\text{mod } p)$

Statement 2: If p divides $(G: H)$, then $N[H] \neq H$

A. Both 1 and 2 are true

B. 1 is true and 2 is false

C. 1 is false and 2 is true

D. Both 1 and 2 are false

48. If G is a group containing normal subgroups H and K such that $H \cap K = \{e\}$ and $H \vee K = G$, then

A. G is equal to HK

B. G is isomorphic to $H \times K$

C. Both of the above

D. None of the above

49. Decomposition of D_3 into conjugate classes is

A. $\{\rho_0\}, \{\rho_1, \rho_2\}, \{\mu_1\}, \{\mu_2, \mu_3\}$

B. $\{\rho_0, \rho_1, \rho_2\}, \{\mu_1, \mu_2, \mu_3\}$

C. $\{\rho_0\}, \{\rho_1\}, \{\rho_2\}, \{\mu_1\}, \{\mu_2\}, \{\mu_3\}$

D. $\{\rho_0\}, \{\rho_1, \rho_2\}, \{\mu_1, \mu_2, \mu_3\}$

50. True statement among the following is

A. Every solvable group is of prime power order

B. Every group of prime power order is solvable

C. Every solvable group is cyclic

D. Every solvable group is abelian

51. Find the number of elements less than and relatively prime to 10.

A. 3

B. 5

C. 4

D. 8

52. Find the number of elements less than and relatively prime to 12.

A. 3

B. 5

C. 4

D. 8

53. Which of the following is a Mersenne prime?

A. 3

B. 5

C. 4

D. 8

54. Which of the following congruence satisfies the Little Theorem of Fermat?

A. $8^{12} \equiv 1 \; mod \; 12$

B. $8^{13} \equiv 1 \; mod \; 13$

C. $8^{12} \equiv -1 \; mod \; 13$

D. $8^{12} \equiv 1 \; mod \; 13$

55. The solution of the linear congruence $2x \equiv 5 \; mod \; 7$ is

A. 6

B. 7

C. 4

D. No Solution

56. The solution of the linear congruence $12x \equiv 27 \; mod \; 18$ is

A. 6

B. 7

C. No Solution

D. 5

57. Which of the following is an example of an Integral Domain?

A. Z_4

B. Z_7

C. Z_6

D. Z_{10}

58. Which of the following is not true?

A. Q is the field of quotients of Z

B. R is the field of quotients of Z

C. R is the field of quotients of R

D. Q is the field of quotients of Q

59. Which of the following is not a field under addition and multiplication?

A. Z

B. R

C. C

D. Q

60. Every field of quotients of Z is isomorphic to

A. Q

B. Z

C. R

D. C

61 Consider the evaluation homomorphism $\varphi_2 : F[x] \to E$ where $F = E = C$. Then $\varphi_2(x^2 + 3)$ is

A. 2

B. 3

C. 1

D. 7

62. Consider the evaluation homomorphism $\varphi_i : F[x] \to E$ where $F = E = C$.

Then $\varphi_i(2x^3 - x^2 + 3x + 2)$ is

A. i

B. i+3

C. 5i+3

D. 4

63. Consider the evaluation homomorphism $\varphi_2 : F[x] \to E$ where $F = Q, E = R$.

Then $\varphi_2(x^2 + x - 6)$ is

A. 2

B. 6

C. 0

D. -2

64. Consider the evaluation homomorphism $\varphi_3 : F[x] \to E$ where $F = E = Z_7$.

Then $\varphi_3[(x^4 + 2x)(x^3 - 3x^2 + 3)]$ is

A. 8

B. 4

C. 6

D. 2

65. Zeros of the polynomial $x^2 + 1$ inZ_2

A. 0

B. 1

C. 0 and 1

D. 2

66. Zeros of the polynomial $x^3 + 2x + 2$ inZ_7

A. 5

B. 2 and 3

C. 2and 6

D. 6

67. Zeros of the polynomial $x^5 + 3x^3 + x^2 + 2x$ inZ_5.

A. 1

B. 3

C. 0 and 4

D. 2 and 4

68. If D is an integral domain, then D[x] is

A. An integral domain

B. A field

C. A finite field

D. Not an integral domain

69. If F is a field, then F[x] is

A. An integral domain

B. A field

C. A finite field

D. Not an integral domain

70. Find the polynomial which is irreducible over Q

A. $x^2 + 3$

B. $x^2 + 5x + 4$

C. x^4

D. $x^2 - 1$

71. Find the polynomial which is reducible overQ

A. $x^2 + 3$

B. $x^2 + 5x$

C. $x^4 - 2$

D. $x^2 + 1$

72. The polynomial $x^2 - 2$ is irreducible over

A. Q

B. R

C. C

D. Z_2

73. The polynomial $x^2 - 2$ is reducible over

A. Q

B. R

C. Z

D. Z_2

74. The polynomial $x^2 + 1$ is irreducible over

A. Q

B. R

C. Z

D. Z_2

75. Which of the following is not true.

A. $x - 2$ is irreducible over Q

B. $3x - 6$ is irreducible over Q

C. $x^2 - 3$ is irreducible over Q

D. $x^2 - 1$ is irreducible over Q

76. A homomorphism of A into itself is called ……………………..

A. Automorphism

B. Endomorphism

C. Isomorphism

D. Monomorphism

77. Wedderburn's theorem states that

A. Every non empty set of positive integers contains a smallest member

B. Every group is isomorphic to a permutation group

C. For any finite group G, the order of subgroup H divides the order of a group G

D. Every finite division ring is a field

78. A map $\emptyset$ of a ring R into a ring R' with $\emptyset(a+b) = \emptyset(a) + \emptyset(b)$ and $\emptyset(ab) = \emptyset(a)\emptyset(b)$ is called

A. Homomorphism
C. Endomorphism
B. Isomorphism
D. Automorphism

79. A ring homomorphism $\emptyset: R \to R'$ is one-one if and only if

A. $\text{Ker}(\emptyset) = \{1\}$
C. $\text{Ker}(\emptyset) = \{0\}$
B. $\text{Ker}(\emptyset) = \{1,2\}$
D. $\text{Range}\ (\emptyset) = \{0\}$

80. Let R be a commutative ring with unity of prime characteristic p. Then the map $\emptyset_p: R \to R$ defined by $\emptyset_p(a) = a^p$ is a homomorphism called……………………..

A. Projection homomorphism
C. Ring Homomorphism
B. Group Homomorphism
D. Frobenius Homomorphism

81. An element 'a' of a ring R with $a^n = 0$ for some $n \in Z^+$ is called…………………

A. Nilpotent
C. Idempotent
B. Orthogonal
D. Diagonal

82. An additive subgroup N of a ring R such that $aN \subseteq N$ and $Nb \subseteq N$ for every a,b$\in R$ is called…………..

A. Factor ring
C. Ideal
B. Prime ideal
D. Prime field

83. Quaternion is a group of order…………………..

A. 8
C. 7
B. 10
D. 6

84. Which of the following is true.

A. $M_n(F)$ has no divisors of zero for any n and any field F
B. Every non zero element of $M_2(Z_2)$ is a unit
C. The group ring RG of an abelian group G is a commutative ring for any commutative ring R with unity
D. No subring of Quarternions is a field

85. The quaternions of Hamilton are …….

A. Commutative ring
C. Field
B. Integral domain
D. Skew field

86. Let s$R_1, R_2, \dots . R_n$ be rings. For each i, the map $\pi_i: R_1 \times R_2 \times \dots . \times R_n \to R_i$ defined by $\pi_i(r_1, r_2, \dots . r_n) = r_i$ is a homomorphism called

A. Projection homomorphism
B. Frobenius Homomorphism
C. Group Homomorphism
D. Ring Homomorphism

87. Let a map $\emptyset: R \to R'$ be a homomorphism of rings. The subring $\emptyset^{-1}(0') = \{r \in R/\emptyset(r) = 0'\}$ is called

A. Range ($\emptyset$)
B. Kernel($\emptyset$)
C. Factor ring
D. Inverse of $\emptyset$

88. Which of the following is true?

A. An isomorphism of a ring R with a ring R' is a homomorphism $\emptyset: R \to R'$such that $Ker(\emptyset) = \{0\}$
B. The Kernel of a homomorphism $\emptyset$ mapping a ring R into a ring R' is $\{r \in R/\emptyset(r) = 0'\}$
C. An ideal N of a ring R is an additive subgroup of $< R, +>$ such that for all $r \in R, n \in N; rn \in N$ and $nr \notin$ N
D. A ring homomorphism $\emptyset: R \to R'$ carries ideals of R into ideals of R'

89. The nil radical of the ring Z_{12} where nilradical is the collection of all nilpotent elements in a commutative ring R, which is an ideal is…………..

A. {0,3}
B. {0,4}
C. {0,9}
D. {0,6}

90. Which of the following is false?

A. Every maximal ideal of every commutative ring with unity is a prime ideal
B. Q is its own prime subfield
C. Every field contains a subfield isomorphic to a prime field.
D. The prime subfield of C is R.

91. Every ideal of the ring Z is of the form ………………

A. Z+n
B. Z-n
C. Z/n
D. nZ

92. Which of the following is a prime field?

A. Z
B. R
C. Q^c
D. Z_p

93. An ideal N$\neq R$ in a commutative ring R is a prime ideal if for a,b$\in R$

A. $ab \in N \implies a \in N \; and \; b \in N$
B. $ab \in N \implies a \notin N \; and \; b \notin N$
C. $ab \in N \implies a \in N \; or \; b \in N$
D. $ab \in N \implies a \notin N \; and \; b \in N$

94. Which of the following is false?

A. An ideal M of R is maximal if and only if R/M is a field, for a commutative ring R

B. An ideal N of R is prime if and only if R/N is an integral domain, for a commutative ring R.

C. Every maximal ideal of R need not be a prime ideal for a commutative ring R

D. Every maximal ideal of R is a prime ideal for a commutative ring R

95. For the ring RG defined as: RG to be the set of all formal sums; ie.$\sum a_i\, g_i$ for all $a_i \in$ R, $g_i \in G$. Under the Operation $(\sum_{i\in I} a_i g_i) + (\sum_{i\in I} b_i g_i) = \sum_{i\in I}(\sum_{g_j g_k = g_i} a_j b_k)\, g_i$ is a group ring of G over F.. If F is a field, then FG is ……………………..of G over F

A. Group ring

B. Ring groups

C. Group field

D. Field Group

96. Let H be a subring of the ring R. Multiplication of additive cosets defined by $(a + H)(b + H) = ab + H$ is well defined if and only if

A. $ah \in$ H and hb $\in$ H, $\forall$a, b $\in$ R and h $\in$ H

B. $ah \in$ H and hb $\notin$ H , $\forall$a, b $\in$ R and h $\in$ H

C. Neither ah nor hb belongs to H, $\forall$a, b $\in$ R and h $\in$ H

D. $ah \notin$ H and hb $\in$ H, $\forall$a, b $\in$ R and h $\in$ H

97. Let N be an ideal of a ring R. The additive cosets of N forms a ring R/N with the binary operation $(a + N)(b + N) = (a + b) + N$. Then the ring R/N is the ……………………

A. Group ring

B. Ring group

C. Group field

D. Field group

98. If R is a ring with unity and N is an ideal of R containing a unit, then

A. $N \subset R$

B. $N = R$

C. $N \neq R$

D. $N \supset R$

99. Z_n is a field if and only if

A. n is prime

B. $n \in N$

C. n=0

D. $n \in Z$

100. Let p(x) be an irreducible polynomial in F[x]. If p(x) divides r(x)s(x) for $r(x), s(x) \in F[x]$, then

A. neither $p(x)|r(x)$ nor $p(x)|s(x)$

B. either $p(x)|r(x)$ or $p(x)|s(x)$

C. $p(x)|r(x)$ and $p(x)|s(x)$

D. All of the above are true