SECOND SEMESTER MSc MATHEMATICS

ADVANCED TOPOLOGY

OBJECTIVE QUESTIONS

1. If every open cover of a topological space has a finite subcover then the space is

(c) Compact

2. Every map from a compact space into a T_2 space is

(a) Closed

3. A continuous bijection from a compact space onto a Hausdorff space is

(c) Homeomorphism

4. Every continuous one-to-one function from a compact space into a Hausdorff space is

(a) Embedding

5. Which of the following is not true for a compact hausdorff space

(d) Completely regular

6. If a space is regular and Lindeloff then it is

(a) Normal

7. If every open cover of a topological space has a countable sub cover then the space is

(a) Lindeloff

8. In a discrete space, every continuous real valued function is

(b) Constant

9. A subset of R with usual topology is closed and bounded then the subset is

(d) Compact

10. If a space X is regular and second countable then X is

(a) Normal

11. A T_4 Space is

(a) Normal and T_1

12. A T3 space is

(c) Regular and T_1

13. If a space X has the property that for every two mutually disjoint closed subsets A and B of X, there exist a continuous function $f: X \to [0 \quad 1]$ suchthat f(A) = {0} and f(B) = {1} then X is

(a) Normal

14. Which of the following is related to the statement given below.

If a space X has the property that for every two mutually disjoint closed subsets A and B of X, there exist a continuous function $f: X \to [0 \quad 1]$ such that f(A) = {0} and f(B) = {1} then X is normal

(a) Urysohn Lemma

15. Which of the following is not true

(d) $T_4 \Longrightarrow$ Second countable

16. Let $A \subset X$ and $f: A \to Y$ then the function $F: X \to Y$ is the extension of f then which of the following is true

(a) $F(a) = f(a) \forall a \in A$

17. Let $A \subset X$ and $f: A \to Y$ also F and G from X to R be two extensions of f then which of the following is true

(a) $F(a) = G(a), \forall\, a \in \bar{A}$

18. Let A be a closed subset of the normal space X and suppose $f: A \to [-1 \quad 1]$ is a continuous function then there exist a continuous function $F: X \to [-1\ 1]$ such that $F(x) = f(x), \forall\, x \in A$ Which of the following is related to the above statement .

(b)Tietze Extension theorem

19. Let A be a closed subset of the normal space X and suppose $f: A \to (-1 \quad 1)$ is a continuous function then which of the following is true

(a)There exist a continuous function $F: X \to (-1 \quad 1)$ such that $F(x) = f(x) \forall x \in A$

20. Which of the following statements is true

(1) Every regular Lindeloff space is normal

(2) Every regular Lindeloff space is second countable

(3) Every regular second countable space is normal

(4) Every regular second countable space is completely regular

(b) 1 and 3

21. Which of the following statements are true

(1) All T_4 spaces are normal

(2) All T_4 spaces are regular

(3)All T_4 spaces are completely regular

(4) All T_4 spaces are Hausdorff

(d) All of the above

22. The range of a map from a compact space into a Hausdorff space is

(c) Quotient space of the domain

23. X is a Hausdorff space and Y is a compact subset of X then Y is

(b) Closed

24. A topological space has the property that for every closed subset A of X , every continuous real valued function on A has a continuous extension to X, then X is

(d) Normal

25..Let $\{X_i : i \in I\}$ be an indexed family of sets then the cartesian product $\prod_{i\in I} X_i$ is defined as

(b)The set of all functions x from the index set I into $\bigcup_{i\in I} X_i$ such that $x(i) \in X_i$ for all $i \in I$.

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26. Let $\{X_i : i \in I\}$ be an indexed family of sets and let $= \prod_{i\in I} X_i$. Let $J \in i$, then the j th projection function π_j is defined as .

(c) $\pi_j : X \to X_j$ defined by $\pi_j(x) = x(j)$ for $x \in X$.

27. Define a box in $X = \prod_{i\in I} X_i$

(c) A subset B of X of the form $\prod_{i\in I} B_i$ for $B_i \subset X_i, i \in I$.

28. Define a wall in $X = \prod_{i\in I} X_i$

(b) A set of the form $\pi_j^{-1}(B_{j)}$ for some $j \in I$and some $B_j \subset X_j$.

29. Which of the following is truem

b..A subset of $X = \prod_{i\in I} X_i$ is a box iff it is the intersection of family of walls.

30. Which of the following is true

a. A subset of $X = \prod_{i\in I} X_i$ is a large box iff it is the intersection of finitely many walls.

31. For any sets Y, I and J which is true up to a set theoreti$cequivalence$

(c). $(Y^I)^J = Y^{IXJ}$

32. Which of the following is true

(c)The intersection of any family of boxes is a box.

33. Let $\{(X_i, \tau_i): i \in I\}$ be an indexed collection of toplogical spaces and let $X = \prod_{i\in I} X_i$ and for each $i \in I$, π_i be the projection function. Then the product topology on X is

a. The smallest topology on X which makes each projection function continuous.
b. The topology on X which makes each projection function continuous.
c. The largest topology on X which makes each projection function continuous.
d. The strongest topology on X which makes each projection function continuous.

Answer (a)

34. A large box is

a. A box $B = \prod_{i \in I} B_i$ where $B_i = X_i, \forall\, i \in I$.

b. A box $B = \prod_{i \in I} B_i$ where $B_i \neq X_i$, except for some $i \in I$.

c. A box $B = \prod_{i \in I} B_i$ where $B_i = X_i$, except for some finite $i \in I$

d. A box $B = \prod_{i \in I} B_i$ where $B_i \neq X_i, \forall\, i \in I$

Answer (c)

35. A cube is

a. $[0,1]^I$, where I is some set.

b. $[0,1]$

c. $[a,b]X[c,d]$

d. $[0,1]^{[0,1]}$

Answer (a)

36 . A Hilbert cube is

a. $[0,1]^I$, where I is denumerable.

b. $[0,1]^I$, where I is some set.

c. $[0,1]^H$

d. $[0,1]$

Answer (a)

37 . Which of the following is true

a. The projection function are continuous and open

b. The projection function are continuous and closed

c. The projection function are open and one to one

d. The projection function are one to one and closed

Answer (a)

38 . Let $\boldsymbol{\tau}$ be the product topology on the set $X = \prod_{i \in I} X_i$,where $\{(X_i, \tau_i): i \in I\}$ is an indexed collection of topological spaces. Then standard sub base for the product topology $\boldsymbol{\tau}$ is

a. The family of all subsets of the form $\prod_{i \in I} V_i$ for $V_i \in \tau_i, i \in I$.

b. The family of all subsets of the form $\pi_i(V_{i)}$ for $V_i \in \tau_i, i \in I$.

c. The family of all subsets of the form ${\pi_i}^{-1}(V_{i)}$ for $V_i \in \tau_i, i \in I$.

d. The family of all subsets of the form $\prod_{i \in I} V_i$.

Answer (c)

39 . Let $\boldsymbol{\tau}$ be the product topology on the set $X = \prod_{i\in I} X_i$,where $\{(X_i, \tau_i): i \in I\}$ is an indexed collection of topological spaces. Then standard base for the product topology $\boldsymbol{\tau}$ is

a. The family of all walls all of whose sides are open in the respective spaces.
b. The family of all boxes all of whose sides are open in the respective spaces.
c. The family of all large boxes all of whose sides are open in the respective spaces.
d. The family of all boxes in X.

Answer (c)

40. If $\{(X_i, \tau_i): i \in I\}$ is an indexed family of spaces having a topological property, the topological product $\prod_{i\in I} X_i$ also has that property then that property is called a

a. Productive property
b. Countably Productive property.
c. Finitely productive property
d. None of these

Answer (a)

41. $T_{0,} T_1, and\ T_2$ are

a. Productive properties
b. Countably Productive properties
c. Finitely productive properties
d. None of these

Answer (a)

42. Regularity and Complete regularity are

a. Productive properties
b. Countably Productive properties
c. Finitely productive properties
d. None of these

Answer (a)

43. Tychonoff property is a

a. Productive property
b. Countably productive property.
c. Finitely productive property

d. None of these.

Answer (a)

44. Connectedness is a

a. Productive property

b. Countably productive property

c. Finitely productive property

d. None of these

Answer (a)

45. Metrisability is a

a. Productive property

b. Countably productive property

c. Finitely productive property

d. None of these

Answer (b)

46. Cantor discontinuum is denoted by

a. Z_2

b. Y^I

c. ${Z_2}^I$

d. None of these

Answer (c)

47. Let S be a sub base for a topological space X. If or each $V \in S$ and for each $x \in V$, there exist a continuous function $f: X \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for all y not in V. Then

a. X is regular

b. X is Tychonoff

c. X is completely regular.

d. None of these

Answer (c)

48. For any cardinal numbers α, β, γ, which one is true

a. $(\alpha^{\beta})^{\gamma} = \alpha^{\beta+\gamma}$

b. $(\alpha^{\beta})^{\gamma} = \alpha^{\beta\gamma}$

c. $(\alpha^{\beta})^{\gamma} = \alpha^{\beta-\gamma}$

d. None of these

Answer (b)

49. Let $\{\alpha_i : i \in I\}$ be an indexed family of cardinal numbers . Then the cardinal number of the product set $\prod_{i\in I} X_i$,where for each $i \in I$, X_i is asset of cardinality α_i is

a. $\sum_{i\in I} \alpha_i$

b. $\prod_{i\in I} \alpha_i$

c. $\bigcup_{i\in I} \alpha_i$

d. None of these

Answer (b)

50. Consider the following two statements

(1) Each coordinate space of a topological product is completely regular then the product space is completely regular

(2) A product of topological spaces is completely regular then each coordinate space is completely regular

Which of the following is true

(a) 1 is true (b) 2 is true (c) Both are true (d) Both are false

Answer (c)

51. Consider the following statements

(1) A topological product of spaces is Tychonoff then each coordinate space is Tychonoff

(2)Each coordinate space of a topological product is Tychonoff then the product space is not Tychonoff

Which of the following is true

(a) 1 is true (b) 2 is true (c) Both are true (d) Both are false

Answer (a)

52. Choose the correct option from the following statements

(1) A topological product of spaces is connected then each coordinate space is not connected

(2)Each coordinate space of a topological product is connected then the product space is not connected

(a) 1 is true (b) 2 is true (c) Both are true (d) Both are false

Answer (d)

53. Let $\{Y_i : i \in I\}$ be an indexed family of sets. Suppose X is a set and let for each $i \in I$, $f_i : X \to Y_i$ be a function. Then the function $e: X \to \prod_{i\in I} Y_i$ defined by $e(x)(i) = f_i(x)$ for $i \in I, x \in X$is know as

(c) Evaluation function

54. An indexed family $\{f_i : i \in I\}$ of functions all defined on the same domain X is said to distinguish points if

(a) For any distinct x, y in X there exist $j \in I$ such that $f_j(x) \neq f_j(y)$

55. The evaluation function of a family of function is one-to-one if and only

(b) That family distinguishes points

56. An indexed family of functions $\{f_i : X \to Y_i : i \in I\}$, where X, Y_i are topological spaces, is said to distinguish points from closed sets in X if

(c) For any $x \in X$ and any closed subset C of X not containing x there exist $j \in I$ such that $f_j(x) \notin \overline{f_j(C)}$ in Y_j.

57. If the family of all continuous real valued functions on a topological space τ distinguish points from closed sets then which among the following is necessarily true?

$(b)\tau$ is completely regular

58. Let $\{f_i : X \to Y_i : i \in I\}$ be a family of functions which distinguishes points from closed sets in X. Then the corresponding evaluation function $e : X \to \prod_{i \in I} Y_i$ is

(a) Open when regarded as a function from X onto $e(X)$

59. Let $\{f_i : X \to Y_i : i \in I\}$ be a family of continuous functions. Then the corresponding evaluation map is an embedding of X into the product space $\prod_{i \in I} Y_i$ if the family

(c) Distinguish points and also distinguish points from closed sets

60. The embedding theorem states that

(b) A topological space is Tychonoff iff it is embeddable into a cube.

61. A space is embeddable in the Hilbert cube if and only if

(c) It is second countable and T_3

62. A second countable space is metrisable if and only if

(d) It is T_3

63. Urysohn's metrisation theorem states that

(d) A second countable space is metrisable iff it is T_3

64. Let X be a topological space. Then a family $\mathcal{U}$ of subsets of X is said to be locally finite if for each $x \in X$, there exists a neighbourhood N of x which intersects

(a) Only finitely many members of $\mathcal{U}$

65. Let X be a topological space. Then a family $\mathcal{V}$ of subsets of X is said to be σ- locally finite if it can be written as the union of

(b) Countably many subfamilies each of which is locally finite

66. A topological space is said to be countably compact if

(b) Every countable open cover of it has a finite sub-cover

67. Which among the following statements is not correct?

(d) Countable compactness is not a weakly hereditary property.

68. For a T_1 countably compact topological space X which among the following is/are true?

(d) All the above

69. For a T_1 countably compact topological space X which among the following is/are true?

(d) All the above

70. Let X be a T_1 topological space such that every sequence in X has a cluster point. Then which among the following is/are true?

(d)All the above

71. Consider the following two statements.

i. A continuous image of a countably compact space is countably compact.

ii. Countable compactness is a weakly hereditary property.

Then choose the correct option.

(a) Both i. and ii. are true

72. Consider the following two statements.

i. Countable compactness is not a weakly hereditary property

ii. If in a T_1 countably compact topological space X every sequence has a cluster point then X is countably compact

Then choose the correct option.

(c) (ii) Is true but (i) is false

73. Consider the following two statements.

i. A metric space is compact if and only if it is countably compact

ii. Every countably compact metric space is second countable

Then choose the correct option.

(a) Both i. an ii. are true

74. Consider the following two statements.

i. Countable compactness is not a weakly hereditary property

ii. If in a T_1 topological space X every sequence has a cluster point then X is countably compact

Then choose the correct option.

(c) ii. is true but i. is false

75. Consider the following two statements.

i. A metric space is compact if and only if it is countably compact

ii. Every continuous real valued function on a countably compact space is bounded and attains its extrema

Then choose the correct option.

(a) Both i. and ii. are true

76. Consider the following two statements.

i. A space is sequentially compact if every sequence in it has a convergent subsequence

ii. A first countable, countably compact space need not be sequentially compact

Then choose the correct option.

(b) i. is true but ii. is false

77. Let X be second countable space. Then choose the right option.

(d) All the above

78.. Consider the following statements

(i) A countably compact metric space is second countable

(ii) A countably compact metric space is compact

Then choose the correct option

(a) Both i and ii are true

79. f is a continuous function from X to R where X is countably compact then f is

(c) Bounded

80 . Which of the following statement is not true

(a) Continuous image of a countably compact space is countably compact

(b) Countable compactness is a hereditary property

(c) Countable compactness is a weakly hereditary property

(d) Acountable compact metric space is second countable

Answer (b)

81. A sequence in a set X is a function;

(a) $f : \mathbb{N} \longrightarrow$ X, where $\mathbb{N}$ is the set of all natural numbers

(b) f : D ⟶ X, where D is a directed set

(c) f : X ⟶ $\mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers

(d) f : X ⟶ D, where D is a directed set

Answer (a)

82 .A directed set is a pair (D, ≥) where D is a non-empty set and ≥ a binary relation on D satisfying;

(c) (i) Transitivity (ii) Reflexivity (iii) For all m, n ϵ D, there exists p ϵ D such that p ≥ m and p ≥ n

83. A net in a set X is a function;

(d) S: D ⟶ X where D is a directed set

84. Which of the following is not an example of a net;

(a) ($\mathbb{N}$, ≥) where $\mathbb{N}$ is the set of all natural numbers and ≥ is the usual ordering on $\mathbb{N}$

(b) (η_x , ≥) where η_x is the neighbourhood system at x and for U, V ϵ η_x, U ≥ V iff V ⊂ U

(c) (D, ≥) where D is the family of all open neighbourhoods of x and for U, V ϵ D, U ≥ V iff U ⊂ V

(d) (D, ≥) where D = $\eta_x \times \eta_y$ and for (U_1, V_1), (U_2, V_2) ϵ D, $(U_1, V_1) \geq (U_2, V_2)$ iff $U_1 \subset U_2$ and $V_1 \subset V_2$

Answer (b)

85. Which of the following is true;

(a) A net is a function with codomain is a directed set

(b) A net is a function with domain is a directed set

(c) Every net converges to some point

(d) Every net has a cluster point

Answer (b)

86. For a topological space X, the limits of all nets in X are unique. Then;

(a) X is a Hausdorff space

(b) X is not a Hausdorff space

(c) There exist *x*, y ϵ X such that U∩V $\neq \phi$ for all U ϵ η_x and V ϵ η_y

(d) For any distinct x, y ϵ X we have U∩V $\neq \phi$ for all U ϵ η_x and V ϵ η_y

Answer (a)

87. D is a directed set and E is an eventual subset of D, then;

(a) For any m ϵ D there exists n ϵ D such that n ≥ m and n ∉ E

(b) Every element of D is in E

(c) There exists m ϵ D such that for all n ϵ D, n ≥ m implies that n ϵ E

(d) There exists m ϵ D such that for all n ϵ D, m ≥ n implies that n ϵ E

Answer (c)

88. S: D ⟶ X is a net in X and S is eventually in a subset A of X, then;

(a) A contains all the terms of S after a certain stage

(b) A contains all the terms of S

(c) A contains no terms of S

(d) For every m ϵ D there exists n ϵ D such that for all n ≥ m and S_n ∉ A

Answer (a)

89. A subset E of a directed set D has the property that for every m ϵ D, there exists n ϵ E such that n ≥ m, then;

(a) E is an eventual subset of D

(b) E is a cofinal subset of D

(c) None of the above is true

(d) Both (a) and (b) are true

Answer (b)

90. A net S: D ⟶ X is said to be frequently in a subset A of X if;

(a) S^{-1}(A) is a cofinal subset of D

(b) S^{-1}(A) is an eventual subset of D

(c) S converges to a point in X

(d) None of the above is true

Answer (a)

91. If a net S converges to x, then for every neighbourhood U of x;

(a) S is eventually in U

(b) S is not eventually in U

(c) S is not frequently in U

(d) None of the above is true

Answer (a)

92. If a net S converges to *x*, then which of the following are true;

(1) *x* is a cluster point of S (2) S is eventually in U for every neighbourhood U of *x*

(3) S is frequently in U for every neighbourhood U of *x*

(d) All are true

93. If *f* is homotopic to *f ′* where *f* is a continuous map and *f ′* is a constant map then;

(a) *f* is a path

(b) *f* is null-homotopic

(c) *f* is path homotopic

(d) None of the above

Answer (b)

94. f : [0, 1] ⟶ X is a continuous function such that *f*(0) = x_0 and *f*(1) = x_1, then;

(a) *f* is a constant path in X

(b) *f* is a path in X from x_1 to x_0

(c) *f* is a path in X from x_0 to x_1

(d) *f* is a path in [0, 1]

Answer (c)

95. . The relation homotopy is;

(a) Reflexive but not symmetric

(b) Reflexive and symmetric but not transitive

(c) Not reflexive

(d) Reflexive, symmetric and transitive

Answer (d)

96. The relation path homotopy is;

(a) Not reflexive

(b) Not symmetric

(c) Both (a) and (b) holds

(d) None is true

Answer (d)

97. f is a path in X from x_0 to x_1 and g is a path in X from x_1 to x_2, then $f * $ g is;

(a) A path in X from x_0 to x_1

(b) A path in X from x_0 to x_2

(c) A path in X from x_2 to x_0

(d) A path in X from x_1 to x_2

Answer (b)

98. f and g are any two maps of a space X into $\mathbb{R}^2$, then;

(a) f and g are homotopic

(b) f and g are not homotopic

(c) f and g are paths in X

(d) None of the above

Answer (a)

99. f and g are paths in X, then the product $f * $ g is a path in X if;

(a) f and g have the same initial point and the same final point

(b) Final point of f is the initial point of g

(c) Initial point of f is the initial point of g

(d) Initial point of f is the final point of g

Answer (b)

100. F is a path homotopy between f and f' then for any s ϵ I;

(a) F(s, 0) = f'(s)

(b) F(s, 0) = x_0

(c) F(s, 0) = f(s)

(d) F(s, 0) = x_1

Answer (c)