M Sc MATHEMATICS

PROGRAM STRUCTURE AND SYLLABUS
2019-20 ADMISSIONS ONWARDS

(UNDER MAHATMA GANDHI UNIVERSITY PGCSS REGULATIONS 2019)

BOARD OF STUDIES IN MATHEMATICS (PG)
MAHATMA GANDHI UNIVERSITY

2019

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ACKNOWLEDGEMENT

The P.G. Board of studies expresses our sincere thanks to the honorable Vice Chancellor of Mahatma Gandhi University, Dr. Sabu Thomas, for the guidance and help extended to us during the restructuring of M.Sc Mathematics syllabus to suit the Credit and Semester System.

We thank Prof. Tomichan Joseph(Syndicate Member, Co-ordinator), for the wholehearted support and for the constant monitoring of the process. His willingness to hear and acknowledge is worth mentioning.

We are highly indebted to Prof..Praveen Kumar V.S.(Syndicate Member, Convener, Syllabus Revision), for his guidance, support as well as for providing necessary information.

The Board of studies thanks the members of M.G. University syndicate for all the help extended to us at various stages of restructuring of this P G Syllabus.

We thank the Registrar of the university, the Academic Section and the Finance Section for extending their service for the smooth completion of the syllabus restructuring. Special thanks are due to the representatives from all the colleges affiliated to M.G. University, who have actively participated in the three day work shop.

The Board of studies acknowledges the contributions from the participants of the workshop. The suggestions and recommendations of the sub groups formed in the workshop have helped us to make the syllabus in the present form.

Table of Contents

M Sc MATHEMATICS Degree Program

(Mahatma Gandhi University Regulations PGCSS2019 from 2019-20 Academic Year)

1. **Aim of the Program**

   Aims and objective:

- To improve the perspective of students on mathematics as per modern requirement.
- To enhance the logical, reasoning, analytical and problem solving skills of students.
- To orient students towards relating Mathematics to applications.
- To help the student build interest and confidence in learning the subject.
- To cultivate a research culture in young minds.
- To encourage students for pursuing higher studies in mathematics.
- To ultimately see that the learning of mathematics becomes more alive, vibrant, relevant and meaningful; a program that paves the way to seek and understand the world around them.
- To motivate students to uphold scientific integrity and objectivity in professional endeavours.

2. **Eligibility for Admissions**
   As per the rules and regulation of PG admission of M G University, 2019

3. **Medium of Instruction and Assessment**
   English

4. **Faculty under which the Degree is Awarded**
   Science

5. **Note on compliance with the UGC Minimum Standards for the conduct and award of Post Graduate Degrees**
   The programme is offered in accordance with the UGC minimum standards for the conduct an award of Post Graduate Degrees. The students have to secure 80 credits to complete the programme successfully.

THE PROGRAM STRUCTURE

Course Code	Title of the Course	Type of the Course	Hours per week	Credits
FIRST SEMESTER				
ME010101	Abstract Algebra	Theory	5	4
ME010102	Linear Algebra	Theory	5	4
ME010103	Basic Topology	Theory	5	4
ME010104	Real Analysis	Theory	5	4
ME010105	Graph Theory	Theory	5	4
SECOND SEMESTER				
ME010201	Advanced Abstract Algebra	Theory	5	4
ME010202	Advanced Topology	Theory	5	4
ME010203	Numerical Analysis with Python	Theory	5	4
ME010204	Complex Analysis	Theory	5	4
ME010205	Measure and Integration	Theory	5	4
THIRD SEMESTER				
ME010301	Advanced Complex Analysis	Theory	5	4
ME010302	Partial Differential Equations	Theory	5	4
ME010303	Multivariate Calculus and Integral Transforms	Theory	5	4
ME010304	Functional Analysis	Theory	5	4
ME010305	Optimization Technique	Theory	5	4
FOURTH SEMESTER				
ME010401	Spectral Theory	Theory	5	4
ME010402	Analytic Number Theory	Theory	5	4
	Elective 1	Theory	5	3
	Elective 2	Theory	5	3
	Elective 3	Theory	5	3
Dissertation				1
Comprehensive Viva				2
Total Credits				80
ME800401	Differential Geometry	Group A	5	3
ME800402	Algorithmic Graph Theory		5	3
ME800403	Combinatorics		5	3
ME810401	Probability Theory	Group B	5	3
ME810402	Operations Research		5	3
ME810403	Coding Theory		5	3
ME820401	Commutative Algebra	Group C	5	3
ME820402	Ordinary Differential Equations		5	3
ME820403	Classical Mechanics		5	3

FIRST SEMESTER COURSES

ME010101	**Abstract Algebra**
ME010102	**Linear Algebra**
ME010103	**Basic Topology**
ME010104	**Real Analysis**
ME010105	**Graph Theory**

ME010101 - ABSTRACT ALGEBRA

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book: **John B. Fraleigh, A First Course in Abstract Algebra, 7th edition, Pearson Education.**

Module 1: Direct products and finitely generated Abelian groups, fundamental theorem, Applications
Factor groups, Fundamental homomorphism theorem, normal subgroups and inner automorphisms.
Group action on a set, Isotropy subgroups, Applications of G- sets to counting.
(Part II – Sections 11, 14, 16 & 17) **(25 hours)**

Module 2: Isomorphism theorems, Sylow theorems , Applications of the Sylow theory.
(Part VII Sections34, 36 & 37) **(25 hours)**

Module 3: Fermat's and Euler Theorems, The field of quotients of an integral domain, Rings of polynomials, Factorisation of polynomials over a field.
(Part IV – Sections 20, 21, 22 & 23) **(20 hours)**

Module 4: Non commutative examples, Homeomorphisms and factor rings, Prime and Maximal Ideals
(Part V – Sections 24, 26 & 27) **(20 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	10	8	4

References:-

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
2. Thomas W. Hungerfor, Algebra (Graduate texts in Mathematics), Springer
3. M. Artin, Algebra, Prentice -Hall of India, 1991
4. N. Jacobson, Basic Algebra Vol. I, Hindustan Publishing Corporation
5. P.B. Bhattacharya, S.K. Jain, S.R. Nagapaul, Basic Abstract Algebra, 2nd edition, Cambridge University Press, Indian Edition, 1997.
6. David S Dummit, Richard M Foote, Abstract Algebra, Third Edition, Wiley.

ME010102 - LINEAR ALGEBRA

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book: **Kenneth Hoffman / Ray Kunze (Second Edition),** ***Linear Algebra*****,** **Prentice-Hall of India Pvt. Ltd., New Delhi, 1992.**

Review : Chapter 1 of text

Module 1: Vector spaces, subspaces, basis and dimension
Co-ordinates, summary of row-equivalence, Computations concerning subspaces
(Chapter 2- 2.1, 2.2, 2.3,2.4 2.5& 2.6 of the text) **(20 hours)**

Module 2: Linear transformations, the algebra of linear transformations, isomorphism, representation of transformations by matrices, linear functional, double dual, transpose of a linear transformation.
(Chapter 3 - 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 & 3.7 of the text) **(25 hours)**

Module 3: Determinants: Commutative Rings, Determinant functions, Permutation and uniqueness of determinants, Additional properties of determinants.
(Chapter 5 - 5.1, 5.2, 5.3 & 5.4 of the text) **(20 hours)**

Module 4: Introduction to elementary canonical forms, characteristic values, annihilatory Polynomials, invariant subspaces, Direct sum Decompositions
(Chapter 6 - 6.1, 6.2, 6.3, 6.4,6.6of the text)
(25 hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	**2**	**2**	**1**
Module II	**3**	**3**	**1**
Module III	**3**	**1**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:

1. Klaus Jonich. Linear Algebra, Springer Verlag.
2. Paul R. Halmos, Linear Algebra Problem Book, The Mathematical Association of America.
3. S. Lang, Algebra, 3rd edition, Addison-Wesley, 1993.
4. K.B. Datta, Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
5. S. Kumaresan, Linear Algebra A Geometrical Approach, Prentice Hall ofIndia, 2000.

ME010103 - BASIC TOPOLOGY

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book : K.D Joshi , Introduction to General Topology , Wiley Eastern Ltd, 1984

Module I : Topological Spaces: Definition of a topological space – Examples of topological spaces-Bases and subbases – subspaces.

(Chapter 4: Sections 1, 2, 3, and 4 of the text) (25 hours)

Module II : Basic concepts: Closed sets and Closures – Neighbourhoods, Interior and Accumulation points – Continuity and Related Concepts – Making functions continuous , Quotient spaces

(Chapter 5: Section 1;1. To 1.7 , Section 2; 2.1 to 2.10 and 2.13, Section3; 3.1 to 3.11, Theorem 3.2 condition 4 (25 hours)

Module III : Spaces with special properties :- Smallness conditions on a space, Connectedness

(Chapter 6 : Section 1; 1.1 to 1.16, Section 2; 2.1 to 2.15 (20 hours)

Module IV : Spaces with special properties :- Local connectedness and Paths

Separation axioms:- Hierarchy of separation axioms

(Chapter 6 : Sections 3.1 to 3.8, Chapter 7 : Sections 1.1 to 1.17(20 hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	10	8	4

Reference:

1) George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company,1963
2) I.M. Singer &J.A. Thorpe ,Lecture Notes on Elementary Topology &Geometry, Springer Verlag 2004

ME010104 - REAL ANALYSIS

5 Hours/Week (Total Hours : 90) **4 Credits**

Text 1: **Tom Apostol, Mathematical Analysis (Second edition), Narosa Publishing House.**

Text 2: **Walter Rudin, Principles of Mathematical Analysis (Third edition), McGraw Hill Book Company, International Editions.**

Module 1: **Functions of bounded variation and rectifiable curves**

Introduction, properties of monotonic functions, functions of bounded variation, total variation, additive property of total variation, total variation on(a, x) as a functions of x, functions of bounded variation expressed as the difference of increasing functions, continuous functions of bounded variation, curves and paths, rectifiable path and arc length, additive and continuity properties of arc length, equivalence of paths, change of parameter.

(Chapter 6, Section: 6.1 - 6.12. of Text 1) **(20 hours.)**

Module 2: **The Riemann-Stieltjes Integral**

Definition and existence of the integral, properties of the integral, integration and differentiation, integration of vector valued functions.

(**Chapter 6 - Section 6.1 to 6.25 of Text 2)** **(20 hours.)**

Module 3: **Sequence and Series of Functions**

Discussion of main problem, Uniform convergence, Uniform convergence and Continuity, Uniform convergence and Integration, Uniform convergence and Differentiation.

(Chapter 7 Section. 7.1 to 7.18 of Text 2) **(25 hours.)**

Module 4: **Weierstrass Approximation &Some Special Functions**

Equicontinuous families of functions, the Stone - Weierstrass theorem, Power series, the exponential and logarithmic functions, the trigonometric functions, the algebraic completeness of complex field.

(Chapter 7 – Sections 7.19 to 7.27, Chapter 8 - Section 8.1 to 8.8 of Text 2)
(25 hours.)

Sections 3.1 to 3.8, Chapter 7 : Sections 1.1 to 1.17(20 hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:-

1. Robert G. Bartle Donald R. Sherbert, Introduction to Real Analysis, 4th Edition, John Wiley and Sons, New York.
2. Gerald B. Folland, Real Analysis: Modern Techniques and Their Applications, 2nd Edition, WeilyInterscience Publication, John Wiley and Sons, New York.
3. Royden H.L, Real Analysis, 2nd edition, Macmillan, New York.
4. Kenneth A. Ross, Elementary Analysis - The Theory of Calculus Second Edition, Springer International
5. ShantiNarayan & M.D. Raisinghania, Elements of Real Analysis, 7th Edition, S. Chand Publishing, New Delhi

ME010105 – GRAPH THEORY

5 Hours/Week (Total Hours : 90) 4 Credits

Text : **R. Balakrishnan and K. Ranganathan , A Text book of Graph Theory, Second edition Springer.**

Module I: Introduction, _Basic concepts. Sub graphs. Degrees of vertices. Paths and Connectedness, Automorphism of a simple graph, line graphs, Operations on graphs, Graph Products.

Directed Graphs : Introduction, basic concepts and tournaments.

(Chapter 1 Sections 1.1 – 1.7(Upto 1.7.3 including) 1.8, 1.9)

(Chapter 1 Sections 2.1, 2.2, 2.3) (20Hours)

Module II: Connectivity : Introduction, Vertex cuts and edge cuts, connectivity and edge connectivity, blocks, Cyclical edge Connectivity of a graph.

Trees; Introduction, Definition, characterization and simple properties, centres and cancroids, counting the number of spanning trees, Cayley's formula. Applications

(Chapter 3 Sections 3.1, 3.2 , 3.3, 3.4 and 3.5)

(Chapter 4 Sections4.1, 4.2, 4.3, 4.4 (Up to 4.4.3 including) and 4.5, 4.7)

(25Hours)

Module III: Eulerian and Hamiltonian Graphs: Introduction, Eulereian graphs, Hamiltonian Graphs, Hamiltonian around' the world' game

Graph Colorings: Introduction, Vertex Colorings, Applications of Graph Coloring, Critical Graphs, Brooks' Theorem

(Chapter 6 Sections 6.1, 6.2 and 6.3)

(Chapter 7 Sections 7.1, 7.2 and7.3(Up to 7.3.1 including) (20Hours)

Module IV: Planarity: Introduction, Planar and Nonplanar Graphs, Euler Formula and Its Consequences, K_5 and $K_{3,3}$ are Nonplanar Graphs, Dual of a Plane Graph, The Four-Color Theorem and the Heawood Five-Color Theorem .

Spectral Properties of Graphs: Introduction, The Spectrum of a Graph, Spectrum of the Complete Graph Kn, Spectrum of the Cycle Cn,

(Chapter 8 Sections 8.1, 8.2 , 8.3, 8.4, 8.5 and 8.6)

(Chapter 11 Sections 11.1, 11.2 , 11.3 and 11.4) (25Hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:

1. John Clark and Derek Allan Holton, A First Look at Graph Theory, Allied Publishers.

2. Douglas B West, Introduction to Graph Theory, Prentice Hall of India.

3. Sheldon Axler, Linear algebra done right, Second edition ,Springer.

SECOND SEMESTER COURSES

ME010201	**Advanced Abstract Algebra**
ME010202	**Advanced Topology**
ME010203	**Numerical Analysis with Python**
ME010204	**Complex Analysis**
ME010205	**Measure and Integration**

ME010201 - ADVANCED ABSTRACT ALGEBRA

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book : **John B. Fraleigh, A First Course in Abstract Algebra, 7th edition, Pearson Education.**

Module 1: Introduction to extension fields, algebraic extensions, Geometric Constructions Finite fields.
(Part VI – Section 29, 31 – 31.1 to 31.18, 32, 33 of the text) **(20 hours)**

Module 2: Unique factorization domains, Euclidean domains. Gaussian integers and multiplicative norms
(Part IX – Sections 45,46 & 47 of the text) **(20 hours)**

Module 3: Automorphism of fields, the isomorphism extension theorem , Splitting fields.
(Part X – Sections 48 & 49, 50 of the text) **(25 hours)**

Module 4: Separable extensions, Galois Theory, Illustrations of Galois Theory, Cyclotomic Extensions. (mention the insolvability of the quintic)
(Sections 51, 53, 54, 55 - 55.1 to 55.6 of the text) **(25 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:-

1. David S Dummit, Richard M Foote, Abstract Algebra, Third Edition, Wiley.
2. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.
3. M. Artin, Algebra, Prentice -Hall of India, 1991.
4. Charles Lanski, Concepts in Abstract Algebra, American Mathematical Society, 2004.
5. Klaus Jonich. Linear Algebra, Springer Verlag.
6. Paul R. Halmos, Linear Algebra Problem Book, The Mathematical Association of America.
7. S. Lang, Algebra, 3rd edition, Addison-Wesley, 1993.
8. K.B. Datta, Matrix and Linear Algebra, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
9. Roger A. Horn, Charles R. Johnson, Matrix Analysis, Second Edition, Cambridge University press.

ME010202 - ADVANCED TOPOLOGY

5 Hours/Week (Total Hours : 90) **4 Credits**

Text 1 : **K.D Joshi , Introduction to General Topology , Wiley Eastern Ltd, 1984**

Text 2 : **James R. Munkres , Topology(second edition) , Pearson**

Module I : Separation axioms:- Compactness and Separation axioms , The Urysohn Characterisation of normality –Tietze Characterisation of normality .
(Chapter 7: Sections 2; 2.1 to 2.10 Section 3; 3.1 to 3.6 – Proof of Lemma 3.4 excluded Section 4; 4.1 to 4.7 of text 1) (20hours)

Module II : Products and Co-products:- Cartesian products of families of sets – The product topology -Productive properties.
(Chapter 8 : Section 1; 1.1 to 1.9 Section 2; 2.1 to 2.8 , Section 3 – 3.1 to 3.6 of text 1) (25hours)

Module III : Embedding and Metrisation;- Evaluation functions into products – Embedding lemma and Tychonoff Embedding – The Urysohn Metrisation Theorem
(Chapter 9: Section 1; 1.1 1.5, Section 2; 2.1 to 2.5 Section, 3; 3.1 to 3.4 Variation of compactness **(Chapter 11:Sections 1.1 to 1.11 of text 1)**
(25 hours)

Module IV : Definition and convergence of nets, Homotopy of paths.
(Chapter 10: Section 1 of text 1; Chapter 9 : Section 1 of text 2) (20hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:

3) George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company,1963
4) I.M. Singer &J.A. Thorpe ,Lecture Notes on Elementary Topology &Geometry, Springer Verlag 2004

ME010203 Numerical Analysis with Python3

5 Hours/Week (Total Hours : 90) **4 Credits**

Text 1	**Jason R Brigs , Python for kids – a playful introduction to programming, No Starch Press**
Text 2	**Amit Saha, Doing Math with Python, No Starch Press, 2015.**
Text 3	**Jaan Kiusalaas, Numerical Methods in Engineering with Python3, Cambridge University Press.**

Though any distribution of Python 3 software can be used for practical sessions, to avoid difficulty in getting and installing required modules like numpy, scipy etc, and for uniformity, the Python3 package *Anaconda 2018.x* (https://www.anaconda.com/distribution/#download-section) may be installed and used for the practical sessions. However, a brief introduction on how to use Python IDLE 3 also should be given.

BASICS OF PYTHON

Before going into mathematics programming part, an introduction to Python should be given.No questions should be included in the end semester examination from this unit. Internal examinations may test the knowledge of concepts from this section.

From Text 1, Chapter 2 full – calculations and variables,

Chapter 3 – creating strings, lists are more powerful than strings, tuples,

Chapter 5- If statements, if-then-else statements, if and elif statements, combining conditions, the difference between strings and numbers,

Chapter 6 – using for loops, while we are talking about looping,

Chapter 7 – using functions, parts of a function, using modules

Chapter 9 – The functions abs, float, int, len, max, min, range, sum

From Text 2 Chapter 1 - section complex numbers

Unit I -

Module I : Defining Symbols and Symbolic Operations, Working with Expressions, Solving Equations and Plotting Using SymPy, problems on factor finder, summing a series and solving single variable inequalities

Chapter 4 - From text 2

Unit II

Module II : Finding the limit of functions, finding the derivative of functions, higher-order derivatives and finding the maxima and minima and finding the integrals of functions are to be done. in the section programming challenges, the following problems - verify the continuity of a function at a point, area between two curves and finding the length of a curve

Chapter 7 from text 2

Unit III

Module III : Interpolation and Curve Fitting - Polynomial Interpolation - Lagrange's Method, Newton's Method and Limitations of Polynomial Interpolation,

Roots of Equations - Method of Bisection and Newton-Raphson Method.

Chapter 3, sections 3.1, 3.2 Chapter 4, sections 4.1, 4.3, 4.5 From Text 3, -

Unit IV-

Module III : Gauss Elimination Method (excluding Multiple Sets of Equations), Doolittle's Decomposition Method only from LU Decomposition Methods

Numerical Integration, Newton-Cotes Formulas, Trapezoidal rule, Simpson's rule and Simpson's 3/8 rule.

Chapter 2, sections 2.2, 2.3 , Chapter 6, sections 6.1, 6.2 From Text 3.

1. Instead of assignments, a practical record book should be maintained by the students. Atleast 15 programmes should be included in this record book.
2. Internal assessment examinations should be conducted as practical lab examinations by the faculty handling the paper.
3. End semester examination should focus on questions including concepts from theory and programming. However, more importance should be given to theory in the end semester examinations as internal examinations will be giving more focus on programming sessions.

Question Paper Pattern

	Section A	**Section B**	**Section C**
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References

1. A primer on scientific programming with python, 3rd edition, Hans Petter Langtangen, Springer
2. Vernon L. Ceder, The Quick Python Book, Second Edition, Manning.
3. NumPy Reference Release 1.12.0, Written by the NumPy community. (available for free download at https://docs.scipy.org/doc/numpy-dev/numpy-ref.pdf)
4. S. D. Conte and Carl de Boor, Elementary Numerical Analysis – An algorithmic approach, Third Edition, McGraw-Hill Book Company.
5. S.S. Sastry, Introductory Methods of Numerical Analysis, Fifth Edition, PHI.

ME010204 COMPLEX ANALYSIS

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book : **Lars V. Ahlfors, Complex Analysis, Third edition, McGraw Hill Internationals**

Module-1 The spherical representation of complex numbers , Riemann Sphere, Stereographic projection, Distance between the stereographic projections

Elementary Theory of power series,Abel's Theorem on convergence of the power series, Hadamard's formula, Abel's limit Theorem

Arcs and closed curves, Analytic functions in regions, Conformal mappings, Length and area ,Linear transformations , The cross ratio, Symmetry, Oriented circles, Families of circles.

Chapter – 1 Section .Chapter – 2Sections.2.1 to 2.5,Chapter – 3 Sections 2.1, 2.2, 2.3,2.4and ,3.1 to 3.4 of the text **(25 hours)**

Module-2 Fundamental theorems on complex integration: line integrals, rectifiable arcs, line integrals as functions of arcs, Cauchy's theorem for a rectangle, Cauchy's theorem in a disk,

Cauchy's integral formula: the index of a point with respect to a cloud curve, the integral formula.

(Chapter 4 – Sections 1 , 2..1 and 2.2 of the text.) **(20 hours.)**

Module-3 Higher derivatives. Differentiation under the sign of integration, Morera's Theorem, Liouville's Theorem, Fundamental Theorem, Cauchy's estimate

Local properties of analytical functions: removable singularities, Taylor's theorem, zeroes and poles,Weirstrass Theorem on essential singularity, the local mapping, the maximum principle.Schwarz lemma

Chapter-4 sectins 2.3, 3.1,3.2,3.3, and 3.4 of the text **(20 hours)**

Module-4 The general form of Cauchy's theorem: chains and cycles, simple connectivity, homology, general statement of Cauchy's theorem, proof of Cauchy's theorem, locally exact differentiation, multiply connected regions

Calculus of Residues: the residue theorem, the argument principle, evaluation of definite integrals.

Chapter-4 Sections 4 and 5 of the text **(25 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:-

1. Chaudhary. B, The elements of Complex Analysis, Wiley Eastern.
2. Cartan. H (1973), Elementary theory of Analytic functions of one or several variable, Addison Wesley.
3. Conway .J.B, Functions of one Complex variable, Narosa publishing.
4. Lang. S, Complex Analysis, Springer.
5. H.A. Priestly, Introduction to Complex Analysis, Clarendon press, Oxford, 1990

ME010205 - MEASURE THEORY AND INTEGRATION

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book: **H. L. Royden, P.M. Fitzpatrick, Real Analysis Fourth Edition, Pearson Education**

Module 1: Lebesgue Measure: Introduction, Lebesgue outer measure, The σ algebra of Lebesgue mesurable sets, Outer and inner approximation of Lebesgue measurable sets , Countable additivity, continuity and Borel-Cantelli Lemma - Non measureable sets - The Canter set and Canter Lebesgue function

Chapter 2; Sections 2.1 to 2.7 **(25 Hours)**

Module 2: Lebesgue Measurable Functions and Lebesgue Integration: Sums, products and compositions – Sequential pointwise limits and simple approximation – The Riemann Integral – The Lebesgue integral of a bounded measurable function over a set of finite measure – The Lebesgue integral of a measurable non negative function – The general Lebesgue integral.

Chapter 3; Sections 3.1 to 3.2, Chapter 4; Sections 4.1 to 4.4(25 Hours)

Module 3: General Measure Space and Measureable Functions: Measures and measurable sets – Signed Measures: The Hanh and Jordan decompositions – The Caratheodory measure induced by an outer measure – Measureable functions

Chapter 17; Sections 17.1 to 17.3, Chapter 18; Section 18.1 upto corollory 7 **(20 Hours)**

Module 4: Integration over General Measure Space and Product Measures: Integration of non negative meaurable functions – Integration of general mesurable functions – The Radon Nikodym Theorem – Product measure: The theorems of Fubini and Tonelli

Chapter 18; Sections 18.2 to 18.4, Chapter 20; Section 20.1 **(20 Hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	10	8	4

References:

1. G. de Barra : Measure Theory and integration , New Age International (P) Ltd., New DelhiNew Age International (P) Ltd., New Delhi,

2.Halmos P.R, Measure Theory, D.vanNostrand Co.

3. P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd., New Delhi, 1986(Reprint 2000).

4. R.G. Bartle, The Elements of Integration, John Wiley & Sons, Inc New York, 1966.

THIRD SEMESTER COURSES

ME010301	**Advanced Complex Analysis**
ME010302	**Partial Differential Equations**
ME010303	**Multivariate Calculus**
ME010304	**Functional Analysis**
ME010305	**Optimization Technique**

ME010301 - Advanced Complex Analysis

5 Hours/Week (Total Hours : 90) **4 Credits**

Text book : **Complex Analysis – Lars V. Ahlfors (Third Edition), McGraw Hill Book Company**

Module 1: Harmonic Functions – Definitions and Basic Properties, The Mean-Value Property, Poisson's Formula, Schwarz's Theorem, The Reflection Principle. A closer look at Harmonic Functions – Functions with Mean Value Property, Harnack's Principle.

The Dirichlet's Problem – Subharmonic Functions, Solution of Dirichlet's Problem (Proof of Dirichlet's Problem and Proofs of Lemma 1 and 2 excluded)

(Chapter 4 : Section 6: 6.1 - 6.5, Chapter 6 : Section 3 : 3.1 - 3.2 , Section 4 : 4.1 - 4.2)

Module 2: Power Series Expansions – Weierstrass's theorem, The Taylor Series, The Laurent Series Partial Fractions and Factorization – Partial Fractions, Infinite Products, Canonical Products, The Gamma Function. Entire Functions – Jensen's Formula, Hadamard's Theorem (Hadamard's theorem - proof excluded)

(Chapter 5 : Section 1 : 1.1 - 1.3, Section 2 : 2.1 – 2.4, Section 3 : 3.1 – 3.2)

Module 3: The Riemann Zeta Function – The Product Development, The Extension of $\zeta(s)$ to the Whole Plane, The Functional Equation, The Zeroes of the Zeta Function

Normal Families – Normality and Compactness, Arzela's Theorem

(Chapter 5 : Section 4 : 4.1 – 4.4, Section 5 : 5.2 - 5.3)

Module 4: The Riemann Mapping Theorem – Statement and Proof, Boundary Behaviour, Use of the Reflection Principle

The Weierstrass's Theory – The Weierstrass's ρ - function, The functions $\zeta(s)$ and $\sigma(z)$, The Differential Equation

(Chapter 6 : Section 1: 1.1-1.3, Chapter 7 : Section 3 : 3.1 – 3.3)

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:

1. Chaudhary B., The Elements of Complex Analysis, Wiley Eastern.

2. Cartan H., Elementary theory of Analytic Functions of one or several variable, Addison Wesley, 1973.

3. Conway J. B., Functions of one complex variable, Narosa publishing.

4. Lang S., Complex Analysis, Springer.

5. H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, Oxford, 1990.

6. Ponnuswamy S., Silverman H., Complex Variables with Applicatio

ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

5 Hours/Week (Total Hours: 90) **4 Credits**

Text Book: **Ian Sneddon , Elements of Partial Differential Equations, Mc Graw Hill Book Company**

Module 1: Methods of solutions of dx/P = dy/Q = dz/R. Orthogonal trajectories of a system of curves on a surface. Pfaffian differential forms and equations. Solution of Pfaffian differential equations in three variables, Partial differential equations. Origins of first order partial differential equation. **(20 hours)**

Module 2: Linear equations of first order. Integral surfaces passing through a given curve. Surfaces orthogonal to a given system of surfaces. Nonlinear partial differential equation of the first order . Compatible systems of first order equations . Charpits Method. Special types of first order equations. Solutions satisfying given conditions. **(25 hours)**

Module 3: Jacobi' s method The origin of second order equations. Linear partial differential equations with constant coefficients. Equations with variable coefficients. **(20 hours)**

Module 4.: Separation of variables. Non linear equations of the second order . Elementary solutions of Laplace equation. Families of equipotential surfaces. The two dimensional Laplace Equation Relation of the Logarithmic potential to the Theory of Functions. **(25 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:

1. Phoolan Prasad and Renuka Ravindran, Partial Differential Equations, New Age International
2. K Sankara Rao, Introduction to Partial Differential Equations, Prentice Hall of India
3. E T Copson, Partial Differential Equations, S Chand and Co

ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

5 Hours/Week (Total Hours : 90) **4 Credits**

Text 1: **Tom Apostol, Mathematical Analysis, Second edition, Narosa Publishing House.**

Text 2: **Walter Rudin, Principles of Mathematical Analysis, Third edition – International Student Edition.**

Module 1: The Weirstrass theorem, other forms of Fourier series, the Fourier integral theorem, the exponential form of the Fourier integral theorem, integral transforms and convolutions, the convolution theorem for Fourier transforms.

(Chapter 11 Sections 11.15 to 11.21 of Text 1) **(20 hours**.)

Module 2: Multivariable Differential Calculus

The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of complex- valued functions, the matrix of a linear function, the Jacobian matrix, the matrix form of the chain rule. Implicit functions and extremum problems, the mean value theorem for differentiable functions,

(Chapter 12 Sections. 12.1 to 12.11 of Text 1) **(22 hours.)**

Module 3: A sufficient condition for differentiability, a sufficient condition for equality of mixed partial derivatives, functions with non-zero Jacobian determinant, the inverse function theorem ,the implicit function theorem, extrema of real- valued functions of one variable, extrema of real- valued functions of several variables.

Chapter 12 Sections-. 12.12 to 12.13 of Text 1

Chapter 13 Sections-. 13.1 to 13.6 of Text 1 **(28 hours.)**

Module 4: Integration of Differential Forms

Integration, primitive mappings, partitions of unity, change of variables, differential forms.

(Chapter 10 Sections. 10.1 to 10.14 of Text 2) **(20 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:-

1. Limaye Balmohan Vishnu, Multivariate Analysis, Springer.

2. Satish Shirali and Harikrishnan, Multivariable Analysis, Springer

ME010304 - FUNCTIONAL ANALYSIS

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book: **Erwin Kreyszig, Introductory Functional Analysis with applications, John Wiley and sons, New York**

Module 1: Examples, Completeness proofs, Completion of Metric Spaces, Vector Space, Normed Space, Banach space, Further Properties of Normed Spaces, Finite Dimensional Normed spaces and Subspaces, Compactness and Finite Dimension

(Chapter 1 – Sections 1.5, 1.6; Chapter 2 - Sections 2.1 to 2.5)

Module 2: Linear Operators, Bounded and Continuous Linear Operators, Linear Functionals, Linear Operators and Functionals on Finite dimensional spaces, Normed spaces of operators, Dual space

(Chapter 2 - Section 2.6 to 2.10)

Module 3: Inner Product Space, Hilbert space, Further properties of Inner Product Space, Orthogonal Complements and Direct Sums, Orthonormal sets and sequences, Series related to Orthonormal sequences and sets, Total Orthonormal sets and sequences, Representation of Functionals on Hilbert Spaces

(Chapter 3 - Sections 3.1 to 3.6, 3.8)

Module 4: Hilbert-Adjoint Operator, Self-Adjoint, Unitary and Normal Operators, Zorn's lemma, Hahn- Banach theorem, Hahn- Banach theorem for Complex Vector Spaces and Normed Spaces, Adjoint Operators

(Chapter 3 - Sections 3.9, 3.10; Chapter 4 - Sections 4.1 to 4.3, 4.5)

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References

1. Limaye, B.V, Functional Analysis, New Age International (P) LTD, New Delhi, 2004
2. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York, 1963
3. Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi, 1989

4. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt. Ltd, Madras, 1994
5. Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut: 1995-96
6. M. Thamban Nair, Functional Analysis, A First Course, Prentice – Hall of India Pvt. Ltd, 2008
7. Walter Rudin, Functional Analysis, TMH Edition, 1974.

ME010305 OPTIMIZATION TECHNIQUES

5 hours/week (Total Hours : 90) **4 credits**

Text -1 **K.V. Mital and C. Mohan, Optimization Methods in Operation Research and Systems Analysis, 3rd edition.**

Text -2 **Ravindran, Philips and Solberg. Operations Research Principle and Practice, 2nd edition, John Wiley and Sons.**

Module I: LINEAR PROGRAMMING

Simplex Method, Canonical form of equations, Simplex Method (Numerical Example), Simplex Tableau, Finding the first BFS and artificial variables, Degeneracy, Simplex multipliers, Revised simplex method, Duality in LPP, Duality theorems, Applications of Duality, Dual simplex method, Summery of simplex methods.

(Chapter 3; sections: 9 – 21 of text – 1) **(25 hours)**

Module II: INTEGER PROGRAMMING

I.L.P in two dimensional space – General I.L.P. and M.I.L.P problems – cutting planes – remarks on cutting plane methods – branch and bound method – examples –general description – the 0 – 1 variable.

(Chapter 6; sections: 6.1 – 6.10 of text – 1) **(25 hours)**

Module III: GOAL PROGRAMMING , FLOW AND POTENTIALS IN NETWORKS

Goal programming. Graphs- definitions and notation – minimum path problem – spanning tree of minimum length – problem of minimum potential difference – scheduling of sequential activities – maximum flow problem – duality in the maximum flow problem – generalized problem of maximum flow.

(Chapter – 5 & 7 Sections 5.9 & 7.1 to 7.9, 7.15 of text - 1) **(15 hours)**

Module IV: NON- LINEAR PROGRAMMING

Basic concepts – Taylor's series expansion – Fibonacci Search - golden section search– Hooke and Jeeves search algorithm – gradient projection search – Lagrange multipliers – equality constraint optimization, constrained derivatives – non-linear optimization: Kuhn-Tucker conditions – complimentary Pivot algorithms.

(Chapter 11; Sections: 11.1 – 11.7, 11.9- 11.11 of text – 2) **(25 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	2	2	1
Module III	2	2	1
Module IV	3	2	1
Total	10	8	4

Reference:-

1. S.S. Rao, Optimization Theory and Applications, 2nd edition, New Age International Pvt.

2. J.K. Sharma, Operations Research: Theory and Applications, 3rd edition, Macmillan India Ltd.

3. Hamdy A. Thaha, Operations Research – An Introduction, 6th edition, Prentice Hall of India Pvt. Ltd.

FOURTH SEMESTER COURSES

ME010401	**Spectral Theory**
ME010402	**Analytic Number Theory**
	Elective Group A
ME800401	**Differential Geometry**
ME800402	**Algorithmic Graph Theory**
ME800403	**Combinatorics**
	Elective Group B
ME810401	**Probability Theory**
ME810402	**Operations Research**
ME810403	**Coding Theory**
	Elective Group C
ME820401	**Commutate Algebra**
ME820402	**Ordinary Differential Equations**
ME820403	**Classical Mechanics**

ME010401 - SPECTRAL THEORY

5 Hours/Week (Total Hours : 90) **4 Credits**

Text Book: **Erwin Kreyszig, Introductory Functional Analysis with applications, John Wiley and sons, New York**

Module I: Reflexive Spaces, Category theorem(statement only), Uniform Boundedness theorem (applications excluded), Strong and Weak Convergence, Convergence of Sequences of Operators and Functionals, Open Mapping Theorem, Closed Linear Operators, Closed Graph Theorem

(Chapter 4 - Sections 4.6 to 4.9, 4.12, 4.13) **(20 Hours)**

Module 2: Banach Fixed point theorem, Spectral theory in Finite Dimensional Normed Spaces, Basic Concepts, Spectral Properties of Bounded Linear Operators, Further Properties of Resolvent and Spectrum, Use of Complex Analysis in Spectral Theory

(Chapter5 – Section 5.1; Chapter 7 - Sections 7.1 to 7.5) (25 Hours)

Module 3: Banach Algebras, Further Properties of Banach Algebras, Compact Linear Operators on Normed spaces, Further Properties of Compact Linear Operators, Spectral Properties of compact Linear Operators on Normed spaces, Further Spectral Properties of Compact Linear Operators

(Chapter 7 - Sections 7.6, 7.7; Chapter 8 - Sections 8.1 to 8.4)

(25 Hours)

Module 4: Spectral Properties of Bounded Self adjoint linear operators, Further Spectral Properties of Bounded Self Adjoint Linear Operators, Positive Operators, Projection Operators, Further Properties of Projections

(Chapter 9 - Sections 9.1 to 9.3, 9.5, 9.6) **(20 Hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References

1. Limaye, B.V, Functional Analysis, New Age International (P) LTD, New Delhi, 2004
2. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York, 1963
3. Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi, 1989
4. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt. Ltd, Madras, 1994
5. Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut: 1995-96
6. M. Thamban Nair, Functional Analysis, A First Course, Prentice – Hall of India Pvt. Ltd, 2008
7. Walter Rudin, Functional Analysis, TMH Edition, 1974.

ME010402 - ANALYTIC NUMBER THEORY

5 Hours/Week (Total Hours : 90) 4 Credits

Text: **Tom M.Apostol,Introduction to Analytic Number Theory,Springer International Student Edition ,Narosa Publishing House**

Module I: Arithmetic Functions, Dirichlet Multiplication and Averages of Arithmetical functions Arithmetic Functions, Dirichlet Multiplication**:**Introduction,The Möbius function μ(n),The Euler totient function ϕ(n), a relation connecting μ and ϕ, a product formula for ϕ(n),The Dirichlet product of arithmetical functions, Dirichlet inversesand the Möbius inversion formula, The Mangoldt function ∧(n),Multiplicative functions, Multiplicative functions and DirichletMultiplication,The inverse of a completely multiplicative function,The Liouville's function λ(n),The divisor function σ_{α}(n),Generalized convolutions

Averages of Arithmetical functions: Introduction,The big oh notation,Asymptotic equality of functions,Eulers summation formula,Some elementary asymptotic formulas,The average order of d(n),The average order of the divisor functions σ_{α}(n),The average order of ϕ(n),An application to the distribution of lattice points visible from the origin,The average order of μ(n) and of ∧(n),The partial sums of a Dirichlet product, Applications to μ(n) and of ∧(n).

(Chapter 2: sections 2.1 to 2.14, Chapter 3:3.1 to 3.11) (30 hours)

Module II: **Some Elementary Theorems on the Distribution of Prime Numbers** Introduction,Chebyshev's functions ψ(x) and ϑ(x),Relation connecting ϑ(x) and π(x),Some equivalent forms of the prime number theorem,Inequalities for π(n) and P_n ,Shapiro's tauberian theorem,Applications of Shapiro's theorem,An asymptotic formula for the partial sum $\sum_{p\le x}\frac{1}{p}$.

(chapter 4: sections 4.1 to 4.8) (15 hours)

Module III: Congruences: Definitions and basic properties of congruences, Residue classes and complete residue system, Linear congruences,Reduced residue systems and Euler-Fermat theorem,Polynomial congruences modulo p,Lagrange's theorem, Applications of Lagrange's theorem,Simultaneous linear congruences,The Chinese remainder theorem, Applications of the Chinese remainder theorem.

(chapter 5: 5.1 to 5.8) (25 hours)

Module IV: Quadratic Residues,The Quadratic Reciprocity Law and Primitive Roots, Quadratic Residues,The Quadratic Reciprocity Law:Quadratic residues,Legendre's symbol and its properties,evaluation of (-1|p) and (2|p),Gauss' Lemma,The quadratic reciprocity law,Applications of the reciprocity law**.(Chapter 9; 9.1 to 9.6)**

Primitive Roots:The exponent of a number mod m, Primitive roots, Primitive roots and reduced residue systems, The nonexistence of primitive roots mod 2^{α} for α≥3,The existence of primitive root mod p for odd primes p, Primitive roots and quadratic residues.

(chapter 10: 10.1 to 10.5) (20 hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	**3**	**2**	**1**
Module II	**2**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:

1.David M. Burton,Elementary number Theory,Seventh edition,Tata McGraw-Hill Edition 2012

2.Kenneth H.Rosen,Elementary Number Theory and its Applications ,sixth edition,2011

3.Hardy G.H. and Wright E.M.,Introduction to the Theory of Numbers,Oxford,1981

4.Leveque W.J.,Topics in Number Theory,Addison Wesley,1961.

5.J.P.Serre, A Course in Arithmetic, GTM Vol.7,Springer-Verlag,1973.

Elective Group A

ME800401 - DIFFERENTIAL GEOMETRY

5 Hours/Week (Total Hours : 90) **3 Credits**

Text Book: John A. Thorpe, Elementary Topics in Differential Geometry

Module 1: Graphs and level sets, vector fields, the tangent space, surfaces, vector fields on surfaces, orientation.
(Chapters 1 to 5 of the text) **(20 hours)**

Module 2: The Gauss map, geodesics, Parallel transport,
(Chapters 6, 7 & 8 of the text) **(20 hours)**

Module 3: The Weingarten map, curvature of plane curves, Arc length and line integrals
(Chapters 9, 10 & 11 of the text) **(25 hours)**

Module 4: Curvature of surfaces and Parametrized surfaces
(Chapters 12 & 14 of the text). **(25 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	**2**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:-

1. Serge Lang, Differential Manifolds
2. I.M. Siger, J.A Thorpe, Lecture notes on Elementary topology and Geometry, Springer – Verlag, 1967.
3. S. Sternberg, Lectures on Differential Geometry, Prentice-Hall, 1964.
4. M. DoCarmo, Differential Geometry of curves and surfaces.
5. Goursat, Mathematical Analysis, Vol – 1(last two chapters)

ME800402: ALGORITHMIC GRAPH THEORY

5 hours/week (Total Hours : 90) **3 credits**

Text Book: **Gray Chartrand and O.R Oellermann , Applied and Algorithmic Graph Theory, Tata McGraw- Hill Companies Inc**

Module I : **Introduction to Graphs and Algorithms**

What is graph? The degree of a vertex. isomorphic graphs. subgraphs, degree sequences. connected graphs. cutvertices and blocks. special graphs. digraphs. algorithmic complexity. Search algorithms, sorting algorithms. greedy algorithms., representing graphs in a computer.

(Chapter 1 Sections 1.1 to 1.9, Chapter 2 Sections 2.1, 2.2 , 2.3, 2.5 and 2.6 of the text) **(24 hours)**

Module II: **Trees, paths and distances**

Properties of trees, rooted trees. Depth-first search,. breadth – first search, . the minimum spanning tree problem

Distance in a graphs, distance in weighted graphs, .the centre and median of a graph. Activity digraphs and critical paths.

(Chapter 3 sections 3.1 to 3.3.3.4 and 3.5 , Chapter 4 sections 4.1 to 4.4 of the text) **(22 hours)**

Module III: **Networks**

An introduction to networks. the max-flow min-cut theorem. the max-flow min-cut algorithm . Connectivity and edge connectivity . Mengers theorem.

(Chapter 5 sections 5.1 , 5.2 , 5.3 and 5.5 of the text) **(22 hours)**

Module IV: **Matchings and Factorizations**

An introduction to matchings . maximum matchings in a bipartite graph,. Factorizations. Block Designs.

(Chapter 6 sections 6.1 , 6.2 , 6.4 and 6.5 of the text) **(22 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	10	8	4

Reference:-

1. Alan Gibbons, Algorithmic Graph Theory, Cambridge University Press, 1985
2. Mchugh. J.A, Algorithmic Graph Theory, Prentice-Hall, 1990

Golumbic. M, Algorithmic Graph Theory and Perfect Graphs, Academic press.

ME800403 COMBINATORICS

5 Hours/Week (Total Hours : 90) 3 Credits

Text : **Chen Chuan Chong , Koh Khee Meng , Principles and Techniques in Combinatorics., World Scientific Publishing, 2007**

Module I Permutations and combinations: Two basic counting principles, Permutations, Circular permutations, Combinations, The injection and bijection principles, Arrangements and selections with repetitions, Distribution Problems
(Chapter 1 of the text Sections 1.1 – 1.7) (22 hours)

Module II The Pigeonhole Principle and Ramsey numbers: Introduction, The Pigeonhole principle, More examples, Ramsey Type problems and Ramsey numbers, Bounds for Ramsey numbers
(Chapter 3 of the text Sections 3.1 - 3.5) (18 hours)

Module III The Principle of Inclusion and Exclusion: Introduction, The principle, A generalization , Integer solutions and shortest routes, Surjective mappings and Stirling Numbers of second kind, Derangements and A Generalization.
(Chapter 4 of the text Sections 4.1 – 4.6) (25 hours)

Module IV Generating Functions & Recurrence relations: Generating Functions: Ordinary generating functions, Some modeling Problems, Partition of Integers, Exponential generating functions
Recurrence Relations: Introduction, Two examples, Linear homogeneous recurrence relations, General Linear recurrence relations.
(Chapter 5 & Chapter 6 of the text Sections 6.1- 6.4) (25 hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	2	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:-

1. V Krishnamoorthy, Combinatorics theory and applications, E. Hoewood, 1986
2. Hall,Jr, Combinatorial Theory, Wiley- Interscinice, 1998.
3 .Brualdi, R A, Introductory Combinatorics, Prentice Hall,1992

ELECTIVE GROUP B

ME810401 PROBABILITY THEORY

5 Hours/Week (Total Hours : 90) 3 Credits

Text 1: **V.K (2001) *An Introduction to Probability and Statistics*, 2^{nd}Edn, Wiley India (P) Ltd, New Delhi.**

Text 2: **Bhat B.R (1999) *Modern Probability Theory* , 3^{rd}Edn, New Age International (P) Ltd, New Delhi.**

Text 3: **S.C Gupta and V.K Kapoor (2002) *Fundamentals of Mathematical Statistics*, 11^{th}Edn, Sultan Chand & Sons, New Delhi.**

Module 1: Introduction and different approaches to probability, Probability Axioms - Addition rule, Principle of inclusion and exclusion, Bonferroni's inequality, Boole's inequality, Implication rule, Sequence of events and their limits, Conditional Probability, Multiplication rule on Probability, Baye's Theorem, Independence of Events, Borel 0-1 Criterion.

Text Book 1 : - Sections 1.2 , 1.3 (till Remarks 5), 1.5, 1.6

Text Book 2 : -Sections 9.3(b)

Module 2: Random variable, Probability distribution, Discrete and Continuous random variables, Function of a random variable, Expectation and Moments of a random variable, Generating Functions, Moment inequalities – Markov's inequality, Chebychev- Bienayme's inequality, Lyapunov's inequality.

Text Book 1 :- Chapters 2- Section 2.1 to 2.5(till example 7) and Chapter 3 –Section 3.2 except proofs of Theorem 4,5,6 , Section 3.3, Section 3.4)

Module 3: Multiple random variable, Independence of random variables, Covariance and Correlation and moments, Addition and Multiplication theorems on expectation, Cr inequality, Holder's inequality, Cauchy- Schwartz's inequality, Jensen's inequality, Minkowski's inequality, Conditional expectation.

Text Book 1 :Sections 4.2 to 4.3 (till example 6), 4.5 (till theorem 6 including its Corollary's 1 and 2, 4.6.

Text Book 2 :Section 5.3 (c) and (d).

Module 4: Convergence of sequence of random variables – Convergence in law, Convergence in probability, Convergence in r^{th} mean, Convergence almost surely. Weak Law of Large Numbers-Kintchine's Weak Law of Large Numbers, Strong Law of Large Numbers-Kolmogrov strong law of large numbers, Central Limit Theorem- Lindberg- Levy form and Liapunov'sform of Central Limit Theorem .(simple application problems)

Text Book 1 : Section 6.2 (till Theorem 12),For the remaining part of the module reference may be done from any of the Text Books 1, 2 or3.

Question Paper Pattern

	Section A	Section B	Section C
Module I	**3**	**2**	**1**
Module II	**3**	**2**	**1**
Module III	**2**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References

1. S.C Gupta and V.K Kapoor (2002) *Fundamentals of Mathematical Statistics*, 11thEdn, Sultan Chand & Sons, New Delhi
2. Mukhopadhyay, P. (2011) *An Introduction to the Theory of Probability*, World Scientific Publishing Company.
3. Billingsley P. (1985) *Probability and Measure*, Wiley India (P) Ltd.
4. Laha R.G and Rohatgi V.K (1979) *Probability Theory*, Wiley India (P) Ltd.
5. Loeve M (1963) *Probability Theory*, Allied East – West Press.
6. Feller W. (1976) *AnIntroduction to Probability Theory and Its Applications*, Vol. 2, Wiley India (P) Ltd.

ME810402 OPERATIONS RESEARCH

5 Hours/Week (Total Hours : 90) 3 Credits

Text 1: **K.V. Mital and C. Mohan, Optimization Methods in Operations Research and System Analysis, 3rd edition, New Age International Pvt. Ltd..**

Text 2 : **A. Ravindran , Don T. Philips and James J. Solberg., Operations Research Principles and Practice, 2nd edition, John Wiley and Sons.**

Module 1: Dynamic Programming Introduction , Problem 1- Minimum path problem, Problem 2 -Single additive constraint, additively separable return, Problem 3– Single multiplicative constraint, additively separable return, Problem 4- Single additive constraint, multiplicatively separable return, Computational economy in DP , Serial multistage model, Examples of failure ,Decomposition , Backward and forward recursion , Systems with more than one constraints, Applications of D.P to continuous systems.

(Chapter 10; Sections 10.1 – 10.12 of text 1)

Module 2: Continuous time random processes An example, Formal definitions and theory, the assumptions reconsidered, Steady state probabilities, Birth death processes, The Poisson process.

(Chapter 6 ; Sections 6.11 – 6.16 of text 2)

Module 3: Queueing Systems Introduction, An example, General Characteristics, Performance Measures, Relations Among the performance Measures, Markovian Queueing Models, The M/M/1 Model, Limited Queue Capacity, Multiple Servers, An example, Finite Sources.

(Chapter 7; Sections 7.1 –7.11 of text 2)

Module 4: Inventory Models Introduction The classical Economic Order Quantity, A Numerical example, Sensitivity Analysis, Non Zero lead Time, The EOQ. with shortages allowed The Production Lot size (PLS) models ,The Newsboy Problem (a single period model) ,A Lot size reorder point model, Variable lead times, The importance of selecting the right model.

(Chapter 8; Sections: 8.1 – 8.14 of text 2)

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	10	8	4

References:

1. Fundamentals of Queueing Theory, Donald Gross, Carl M. Harris, 3rd edition, John Wiley and Sons.
2. Hamdy A. Taha, Operations Research – An Introduction, 6th edition, Prentice Hall of India Pvt. Ltd.
3. Man Mohan, P.K. Gupta and Kanti Swarup, Operations Research, Sultan Chand and Sons.

ME810403 : CODING THEORY

5 hours/week (Total Hours : 90) **3 Credits**

Text : **Vera Pless 3rd Edition , Introduction to the theory of error coding codes, Wiley Inter Science**

Module I: Introduction Basic Definitions Weight, Maximum Likelihood decoding Synarome decoding, Perfect Codes, Hamming codes, Sphere packing bound, more general facts.
(Chapter 1 & Chapter 2 Sections 2.1, 2.2, 2.3 of the text) **(25 hours)**

Module II: Self dual codes, The Golay codes, A double error correction BCH code and a field of 16 elements.
(Chapter 2 Section 2.4 & Chapter 3 of the text) **(20 hours)**

Module III: Finite fields
(Chapter 4 of the text) **(20 hours)**

Module IV: Cyclic Codes, BCH codes
(Chapter 5 & Chapter 7 of the text) **(25 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	3	2	1
Module III	2	2	1
Module IV	2	2	1
Total	10	8	4

References:-

1. R-Lidi, G. Pliz, Applied Abstract Algebra, Springer Verlag.
2. J.H.Van Lint, Introduction to Coding Theory, Springer Verlag
3. R.E.Blahut, Error- Control Codes, Addison Wesley.

ELECTIVE GROUP C

ME820401: COMMUTATIVE ALGEBRA

5 hours/week (Total Hours : 90) **3 credits**

Text Book : **Gregor Kemper, A Course in Commutative Algebra, Springer, ISSN0072-5285, ISBN978-3-642-03544-6**

Module I: **The Algebra-Geometry Lexicon – Hilbert's Nullstellensatz**

Maximal ideals, Jacobson Rings, Coordinate Rings, Simple problems.

(Chapter1 Sections 1.1, 1.2 & 1.3 of the text) **(25 hours)**

Module II: **Noetherian and Artinian Rings.**

The Noether and Artin Properties for Rings and Modules, Notherian Rings and Modules, Simple problems

(Chapter2 Sections 2.1 & 2.2, of the text) **(20 hours)**

Module III: **The Zariski Topology**

Affine Varieties, Spectra, Noetherian and Irreducible Spaces, Simple problems.

(Chapter 3 Sections 3.1, 3.2 & 3.3 of the text) **(25 hours)**

Module IV: **A Summary of the Lexicon**

True Geometry: Affine Varieties, Abstract Geometry : Spectra , Simple problems

(Chapter 4 Sections 4.1 & 4.2, of the text). **(20 hours)**

Question Paper Pattern

	Section A	Section B	Section C
Module I	3	2	1
Module II	2	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References: -

1. William W. Adams, Phillippe Loustaunau, An Introduction to Grobner bases, Graduate Studies in Mathematics 3, American Mathematical Society, 1994, [117]
2. Michael F Attiyah, Ian Grant Macdonald, Introduction to Commutative Algebra, Addison- Wesley, Reading, 1969[174]
3. Nicolas Bourbaki, General Topology, Chapters 1 – 4, Springer, Berlin, 1993, [117,118,161].

ME820402: Ordinary Differential Equations

5 hours/week (Total Hours : 90) 3 credits

Text : George F. Simmons, Differential Equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill Publishing Company Limited

Module I : Qualitative properties of solutions and Boundary value problems

Oscillations and the sturm separation theorem,the sturm comparison theorem, Eigen values, eigen functions and the vibrating string , Sturm – Liouville problems

(Sections 24, 25, 40 and 43 of the text) (20 hours)

Module II : Some Special Functions of Mathematical Physics

Legendre polynomials, Properties of Legendre polynomials, Bessel`s functions, the gamma function, properties of Bessel functions, Additional properties of Bessel functions

(Sections 44, 45, 46, 47 and Appendix C of the text) (25 hours)

Module III : System of First order Equations and Non linear Equations

General remarks on systems, Linear systems, Homogeneous linear systems with constant coefficients, Nonlinear systems. Volterra`s prey- predator equations, Autonomous systems. The phase plane and its phenonina, Types of Critical points. Stability, Critical Points and stability for linear systems, Stability by Liapunov`s direct method.

(Sections 54 to 61 of the text) (25 hours)

Module IV: The Existence and Uniqueness of Solutions

The method of successive approximations, Picard`s theorem, Systems. The second order linear equation

(Sections 68, 69 and 70 of the text) (20 hours)

Question Paper Pattern

	Section A	Section B	Section C
Module I	2	2	1
Module II	3	2	1
Module III	3	2	1
Module IV	2	2	1
Total	10	8	4

References:-

1. Shepley L. Ross - Differential Equations, 3rd ed., (Wiley India).
2. E.A. Coddington - An Introduction to Ordinary Differential Equation, PHI.
3. W.E. Boyce & R.C. Diprima - Elementary Differential Equations and boundary value Problems, (Wiley India)
4. S. Balachandra Rao & H. Ranuradha – Differential Equation with Applications and Programs (Universities Press)

ME820403 CLASSICAL MECHANICS

5 Hours/Week (Total Hours : 90) **3 Credits**

Text: **L. D. Landau and E. M. Lifshitz- MECHANICS, (Third Edition) (Butter worth – Heinenann)**

Module 1: Generalized coordinates, the Principle of least action, Galileo's relativity principle, the Legrangian for a free particle, Legrangian for a system of particle,energy, momentum, center of mass, angular momentum, motion in one dimension,determination of the potential energy from the period of oscillation, the reduced mass,motion in a central field.

(Section 1 to 9, 11 to 14 of the text) **(25 hours)**

Module 2: Free oscillation in one dimension, angular velocity, the inertia tensor, angular momentum of a rigid body, the equation of motion of a rigid body, Eulerian angle,Euler's equation.

(Section 21, 31 to 36 of the text) **(20 hours)**

Module 3: The Hamilton's equation, the Routhian, Poisson brackets, the action as a functionof the coordinates, Maupertui's principle.

(Section 40 to 44 of the text) **(25 hours)**

Module 4: The Canonical transformation, Liouville's theorem, the Hamiltonian – Jacobi equation, separation of the variables, adiabatic invariants, canonical Variables

(Section 45 – 50 of the text) **(20 hours)**

Question Paper Pattern

	Section A	**Section B**	**Section C**
Module I	**3**	**2**	**1**
Module II	**2**	**2**	**1**
Module III	**3**	**2**	**1**
Module IV	**2**	**2**	**1**
Total	**10**	**8**	**4**

References:-

1. M. G. Calkin, Lagrangian and Hamiltonian Mechanics, Allied
2. Herbert Goldstein, Classical Mechanics, Narosa
3. K C Gupta, Classical Mechanics of particles and Rigid Bodies, Wiley Eastern

3.MODEL QUESTION PAPER PATTERN

QP Code **Reg. No……………………..**

Name …………………..………

M Sc (Mathematics)Degree (C.S.S) Examination, ………

………. Semester

Faculty of Science

Name of Course

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

<u>Section- A</u>

(Answer any **eight** questions. Each question carries a weight of 1)

1.

2.

3.

4.

5.

6.

7.

8.

9.

10. **(8 x 1 = 8)**

<u>Section B</u>

(Answer any **six** questions. Each question carries a weight of 2)

11

12.

13.

14.

15.

16.

17.

18. (**6 x 2 = 12**)

<u>Section C</u>

(Answer any **two** questions. Each question carries a weight of 5.)

19.

20.

21.

22. (**2 x 5 = 10)**

Model Question Papers

QP Code **Reg. No. ………….**

Name ……………..

M Sc (Mathematics)Degree (C.S.S) Examination, ………

Fourth Semester

Faculty of Science

ME010101 ABSTRACT ALGEBRA

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

Section- A

(Answer any **eight** questions. Each question carries a weight of 1)

1. Find all abelian groups, up to isomorphism, of order 16.
2. Find the order of $5+<4>$ in the factor group $\mathbb{Z}_{12}/<4>$
3. If X is a $G-$set, prove that G_x is a subgroup of G for each $x \in X$.
4. Find the kernel of the homomorphism $\phi: \mathbb{Z}_{18} \to \mathbb{Z}_{12}$ where $\phi(1) = 10$
5. Show that every group of order 45 has a normal subgroup of order 9.
6. Give the conjugate classes and the class equation of S_3
7. Find all solutions of the congruence $12x \equiv 27 (\mathrm{mod}\ 18)$
8. Show that $x^4 - 2x^2 + 8x + 1$ is irreducible over $\mathbb{Q}$.
9. Give an example to show that factor ring of an integral domain may be a field.
10. Prove that a field has no proper nontrivial ideals. **(8 x 1 = 8)**

Section B

(Answer any **six** questions. Each question carries a weight of 2)

11. Let H be a subgroup of a group G. Prove that left coset multiplication is well defined by the equation $(aH)(bH) = (ab)H$ if and only if H is a normal subgroup of G.
12. Derive Burnside's formula.
13. Let H be a subgroup of a group $G$and Let N be a normal subgroup of a group G. Prove that $(HN)/N \simeq H/(H \cap N)$
14. Prove that every group of order 255 is abelian and cyclic.
15. Prove that every field L containing an integral domain D contains a field of quotients of D.
16. State and prove Eisenstein Criterion for irreducibility of polynomials.

17. Show that the set $\mathrm{End}(A)$ of all endomorphisms of an abelian group A forms a ring under
homomorphism addition and homomorphism multiplication.
18. Prove that a field F is either of prime characteristic p and contains a subfield isomorphic to $\mathbb{Z}_p$ or of characteristic 0 and contains a subfield isomorphic to $\mathbb{Q}$.

(6 x 2 = 12)

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. a) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.
b) Prove that if m divides the order of a finite abelian group G, then G has a subgroup of order m.
20. a) Let p be a prime and G be a finite group. If p divides $|G|$, prove that G has an element of order p.
b) State and prove Third Sylow theorem.
21. a) State and prove Division Algorithm for $F[x]$.
b) Prove that if G is a finite subgroup of the multiplicative group $(F^*, \cdot)$ of a field F, then G is cyclic.
22. a) Prove that if F is a field, then every ideal in $F[x]$ is principal.
b) Prove that an ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal if and only if $p(x)$ is irreducible over$F$.

(2 x 5 = 10)

QP Code **Reg. No. …………..**

Name ………………

M Sc (Mathematics)Degree (C.S.S) Examination, ………

First Semester

Faculty of Science

ME 010103 BASIC TOPOLOGY

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

<u>Section- A</u>

(Answer any **eight** questions. Each question carries a weight of 1)

1. 1. Define a topological space. Give two different topologies on $\mathcal{R}$.
2. Let $\{\tau_i / i \in I\}$ be an indexed collection of topologies on a set X. Show that $\tau = \bigcap_{i \in I} \tau_i$ is a topology on X.
3. Give two topologies on a topological space which are not comparable. Justify your answer.
4. Show that a subset A of a topological space X is dense in X if and only if for every non-empty open subset B of , $A \cap B = \phi$.
5. Prove that the product topology is the weak topology determined by the projection functions.
6. Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
7. Let $\mathcal{C}$ be a collection of subsets of a space X such that no two members of $\mathcal{C}$ are mutually separated. Prove that $\bigcup_{C \in \mathcal{C}} C$ is connected.
8. Prove that every continuous image of a compact space is compact.
9. Let X be a locally connected space. Prove that the components of a open subsets of X are open in X.
10. Show that regularity is a hereditary property. **(8 x 1 = 8)**

<u>Section B</u>

(Answer any **six** questions. Each question carries a weight of 2)

11. Prove that metrisability is a hereditary property.

12. If a space is second countable, prove that every open cover of it has a countable subcover.
13. For a subset A of a space X, show that $\bar{A} = A \cup A'$.
14. Prove that every open surjective map is a quotient map.
15. Show that every second countable space is first countable.
16. Define weakly hereditary property. Give an example. Prove it..
17. Show that every path connected space is connected.
18. Show that any completely regular space is regular.. (**6 x 2 = 12**)

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. a) Let X be a set, τ a topology on X and S a family of subsets of X. Show that S is a sub-base for τ if and only if S generates τ.

    b) Let (X, τ) be second countable and $Y \subset X$. Show that any cover of Y be members of τ has a countable subcover.
20. Let (X, τ) and (Y, u) be spaces and $f: X \to Y$ a function. Prove that the following statements are equivalent.

    (a) f is continuous

    (b) For all $V \in u, f^{-1}(V) \in \tau$

    (c) There exists a sub-base S for u such that $f^{-1}(V) \in \tau$ for all $V \in S$

    (d) For any closed subset A of Y, $f^{-1}(A)$ is closed in X

    (e) For all $A \subset X$, $f(\bar{A}) \subset \overline{f(A)}$
21. a) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

    b) State and prove Lebesgue covering lemma.
22. a) Prove that in a Hausdorff space, limits of sequences are unique.

    b) Prove that all metric spaces are T_4 (**2 x 5 = 10)**

QP Code **Reg. No. ………….**

Name ……………..

M Sc (Mathematics)Degree (C.S.S) Examination, ………

First Semester

Faculty of Science

ME010105 GRAPH THEORY

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

Section- A

(Answer any **eight** questions. Each question carries a weight of 1)

1. State and prove the first theorem in graph theory and deduce that the number of vertices in a cubic graph is always even.

2. If G_1 and G_2 are two graphs , find the order and size of their Cartesian product in the corresponding terms of G_i, i=1,2.

3. Show that a vertex v cannot be a cut vertex of both a graph and its complement.

4. Construct a graph with vertex connectivity = 2 = edge connectivity.

5. Prove that every tree is bipartite and characterize all complete bipartite graphs which are trees.

6. Prove that an eulerian graph is bridgeless.

7. Prove that the only 3-critical graphs are odd cycles.

8. Prove that every k critical graph is 2-connected.

9. Prove that the number of faces of a planar graph G is independent from the planar drawing of G.

10. Obtain the adjacency spectrum of K_3. **(8 x 1 = 8)**

Section B

(Answer any **six** questions. Each question carries a weight of 2)

11. Prove that the set of all automorphisms on a simple graph G forms a group under the composition of functions.

12. Prove that every diconnected tournament is Hamiltonian.

13. Prove that the vertex connectivity and edge connectivity of a simple cubic graph are equal.

14. Prove that the centre of a tree consists of a single vertex or two adjacent vertices.

15. Prove that a graph G is eulerian if and only if its edges can be partitioned into edge disjoint cycles.

16. State and prove Ores theorem.

17. State and prove Eulers Formula for a connected planar graph and deduce that K_5 is nonplanar.

18. Obtain the spectrum of C_n. (**6 x 2 = 12**)

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. Prove that

    1. A graph G is bipartite if and only if it has no odd cycles.
    2. Prove that the line graph of a graph G is a cycle if and only if G is a cycle.
    3. Prove that every tournament contains a directed Hamiltonian path.

20 . State and prove Cayleys formula by proving all necessary lemmas.

21. State and prove Brookes theorem

22. State the four color conjecture, prove the 6 color and five color theorems.

(2 x 5 = 10)

QP Code **Reg. No. ………….**

Name ……………..

M Sc (Mathematics)Degree (C.S.S) Examination, ………

Third Semester

Faculty of Science

ME 010202 ADVANCED TOPOLOGY

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

Section- A

(Answer any **eight** questions. Each question carries a weight of 1)

1. Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.
2. Let A be a subset of a space X and let $f: A \to \mathcal{R}$ be continuous. Prove that any two extensions of f to X agree on $\bar{A}$.
3. Prove that projection functions are open.
4. Prove that a product of topological spaces is completely regular if and only if each co-ordinate space is so.
5. Define the terms box and wall in a set $X = \prod_{i \in I} X_i$ where $\{X_i / i \in I\}$ is an indexed family of sets.
6. Define the evaluation function of the indexed family of functions. Prove that the evaluation function of a family of functions is one-one if and only if the family distinguishes points.
7. Prove that a topological space is completely regular if and only if the family of continuous real valued functions on it distinguishes points from closed sets.
8. Prove that the evaluation function continuous if and only if f_i each is continuous.
9. Let $S: D \to X$ be a net and F a cofinal subset of S. If ${S}/{F} : F \to X$ converges to a point x in X, prove that x is a cluster point of S.
10. Define (a) Riemann net (b) Homotopy between two maps

    (c) Path homotopy between f and f' **(8 x 1 = 8)**

Section B

(Answer any **six** questions. Each question carries a weight of 2)

11. Prove that a compact subset in a Hausdorff space is closed.
12. State and prove Tietze characterization of normality.

13. Prove that a subset of X is a box if and only if it is the intersection of a family of walls.
14. Let C_i be a closed subset of a space X_i for $i \in I$. Prove that $\prod_{i \in I} C_i$ is closed subset of $\prod_{i \in I} X_i$ with respect to the product topology.
15. Prove that a topological space is a Tychonoff space if and only if it is embeddable into a cube.
16. Prove that a second countable space is metrisable if and only if it is T_3.
17. Prove that a topological space is Hausdorff if and only if limits of all nets in it are unique.
18. Let $(D, \geq)$ be a directed set and E is an eventual subset of D, then prove that E with the restriction of $\geq$ is a directed set. Also prove that $S: D \to X$ converges to x in X if and only if $S/_E : E \to X$ converges to x in X. **(6 x 2 = 12)**

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. State and prove Urysohn lemma.
20. a) Prove that if the product is non-empty, then each co-ordinate space is embeddable in it.
    b) Prove that a topological spaces is regular if and only if each co-ordinate space is regular.
21. a) State and prove embedding lemma.
    b) Prove that a space is embeddable in the Hilbert cube if and only if it is second countable and T_3.
22. a) Define subnet.
    b) Let $S: D \to X$ be a net in a topological space and let $x \in X$. Then x is a cluster point of S if and only if there exists a subnet of S which converges to x in X.

(2 x 5 = 10)

QP Code **Reg. No. …………..**

Name ………………

M Sc (Mathematics)Degree (C.S.S) Examination, ………

Second Semester

Faculty of Science

ME010204 COMPLEX ANALYSIS

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

Section- A

(Answer any **eight** questions. Each question carries a weight of 1)

1. 1. If $\sum a_n z^n$ and $\sum b_n z^n$ have radii of convergence R_1 and R_2 respectively, show that the radius of convergence of $\sum a_n b_n z^n$ is atleast R_1R_2.
2. If $T_1 z = \dfrac{z+2}{z+3}$ and $T_2 z = \dfrac{z}{z+1}$,find T_1T_2z, $T_1^{-1}T_2 z$ and $T_2^{-1}T_1 z$.
3. Prove that an analytic function f(z) degenerates if arg f(z) is a constant.
4. Find the linear transformation which carries 0,i,-I to 1,-1,0 respectively.
5. Prove that $\left|\int_\gamma f(z)dz\right| \le \int_\gamma |f(z)|dz$.
6. Compute $\int_{|z|=2} \dfrac{dz}{z^2-1}$.
7. Prove that if γ lies inside of a circle C, then $n(\gamma, a) = 0$, for all points a, outside C.
8. Compute $\int_{|z|=1} z^{-4} \sin z dz$.
9. Prove that a non constant analytic function maps open sets onto open sets.
10. Prove that reflection carries circles onto circles.

(8 x 1 = 8)

Section B

(Answer any **six** questions. Each question carries a weight of 2)

11. Prove that analytic function f(z) in a region Ω , is conformal at z_0 in Ω if and only if $f'(z_0) \neq 0$.
12. Find the linear transformation which carries the circle $|z| = 2$ into $|z+1| = 1$, the point -2 to the origin and the origin to i.
13. Derive Cauchy's integral formula.
14. State and prove Liouville's theorem.
15. State and prove Weirstrass's theorem for essential singularities.
16. State and prove the theorem on local correspondence.
17. State and prove Cauchy's theorem for a disk.
18. Use Schwarz Lemma to show that $\frac{|f'(z)|}{\left(1-|f(z)|^2\right)} \leq \frac{1}{1-|z|^2}$,where f(z) is analytic in the unit disc and $|f(z)| \leq 1$. **(6 x 2 = 12)**

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. (a) Define cross ratio. Prove that four points z_1,z_2,z_3,z_4 are concyclic if and only if their cross ratio is real

    (b) Prove that a linear transformation carries circles onto circles

    (c)Find the most general linear transformation which maps IzI=R onto itself.

20. (a) State and prove Cauchy,s theorem for a rectangle.

    (b) Discuss the case when there are exceptional points inside the rectangle..
21. (a)State and prove the theorem on higher derivatives.

    (b)Prove that analytic function has derivatives of all orders and all are analytic.

    (c)Derive the Cauchy's estimate for n'th derivative.
22. (a)State and prove the general form of Cauchy's theorem.

    (b)Evaluate $\int_0^\infty \frac{\cos x dx}{x^2+a^2}$; a being real. **(2 x 5 = 10)**

QP Code **Reg. No. …………..**

Name ………………

M Sc (Mathematics)Degree (C.S.S) Examination, ………

Third Semester

Faculty of Science

ME 010301 ADVANCED COMPLEX ANALYSIS

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

Section- A

(Answer any **eight** questions. Each question carries a weight of 1)

19. Define a harmonic function. If f(z) is an analytic function, prove that $\log|f(z)|$ is harmonic.
20. Derive the Taylor series for arc sin z about the origin. Also find the radius of convergence.
21. Define an entire function with an example. What is the connection between the genus and order of an entire function?
22. Find the residue of the Gamma function at the poles $z = -n$.
23. State the Reflection Principle.
24. State Arzela's theorem.
25. If f(z) is analytic, then prove that $\log\left(1+|f(z)|^2\right)$ is sub harmonic.
26. State the Riemann Mapping theorem.
27. Prove that the sum of residues of an elliptic function is zero.
28. Derive the Legendre's relation, $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$.

(8 x 1 = 8)

Section B

(Answer any **six** questions. Each question carries a weight of 2)

29. State and prove Schwarz's theorem.
30. Find the Laurent series for the function $f(z) = \frac{1}{z(z-1)}$ and find the regions where the expansions are valid.

31. Derive the necessary and sufficient condition for the absolute convergence of the infinite product $\prod_{1}^{\infty}(1+a_n)$.

32. Derive the Poisson – Jensen's formula for $\log|f(z)|$.

33. Write a short note on the zeros of Riemann Zeta function.

34. State and prove the boundary behavior theorem for analytic functions.

35. Explain what is meant by Dirichlet's problem. State the theorem behind the Perron's method for solving the problem for arbitrary regions.

36. Derive the differential equation for the Weirstrass' $\wp$ function **(6 x 2 = 12)**

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

37. (a) State and prove Mittag Leffler's theorem of partial fractions.

    (b) Prove that $\pi \cot \pi z = \frac{1}{z} + \sum_{n \neq 0}\left[\frac{1}{z-n} + \frac{1}{n}\right].$

38. (a) Obtain the product and integral representations for the Riemann Zeta function.

    (b) Prove that it can be extended as a meromorphic function in the whole plane with a single simple pole at s=1.

39. Prove that a family $\Im$ is normal if its closure with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty}\delta_k(f,g)2^{-k}$ is compact.

40. Prove that (a) $\zeta(z+u) = \zeta(z) + \zeta(u) + \frac{1}{2}\left[\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}\right].$

    (b) $\wp(z+u) = -\wp(z) - \wp(u) + \frac{1}{4}\left[\frac{\wp'(z)-\wp'(u)}{\wp(z)-\wp(u)}\right]^2$

    (c) $\begin{vmatrix} \wp(z) & \wp'(z) & 1 \\ \wp(u) & \wp'(u) & 1 \\ \wp(z+u) & -\wp'(z+u) & 1 \end{vmatrix} = 0$

(2 x 5 = 10)

QP Code **Reg. No. ………….**

Name ……………..

M Sc (Mathematics)Degree (C.S.S) Examination, ………

Third Semester

Faculty of Science

ME010304 FUNCTIONAL ANALYSIS

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

<u>Section- A</u>

(Answer any **eight** questions. Each question carries a weight of 1)

1. Is $C[a,b]$ a Banach space? Justify your Answer.
2. If X is a finite dimensional vector space, prove that any norm on X is equivalent to any other norm on it.
3. Show that every linear operator on a finite dimensional space is bounded.
4. Show that a finite dimensional vector space is algebraically reflexive.
5. Define Dual space of a normed space. What is the dual of $\mathbb{R}^n$?
6. Give an example of a Normed space which is not an inner product space. Justify.
7. If Y is a closed subspace of a Hilbert space, then prove that $Y = Y^{\perp\perp}$
8. Give an example of $x \in l^2$ such that we have strict inequality occurs in an inner product space
9. Define a reflexive space. Give example of a space which is (a) Reflexive, (b) Non-reflexive.
10. Prove that a bounded linear operator T on a complex Hilbert space H is unitary if and only if T is isometric and surjective.

(8 x 1 = 8)

<u>Section B</u>

(Answer any **six** questions. Each question carries a weight of 2)

11. Prove that every finite dimensional subspace of a Normed space is complete.

12. If X is a Normed space such that the closed unit ball is compact, then prove that X is finite dimensional.
13. Let $T: D(T) \rightarrow Y$ be a bounded linear operator, where $D(T)$ lies in a Normed space X and Y is a Banach space. Prove that T has an extension $\tilde{T}: \overline{D(T)} \rightarrow Y$ where $\tilde{T}$ is a bounded linear operator of norm $\|\tilde{T}\| = \|T\|$
14. Show that the dual space of l^1 is l^∞
15. State and prove Riesz's theorem for functionals on Hilbert spaces.
16. Let H be a Hilbert space. Then prove that if H contains an orthonormal sequence which is total in H, then H is separable.
17. Prove that the adjoint operator $T^\times$ of T is linear, bounded and $\|T^\times\| = \|T\|$
18. Prove that every vector space $X \neq \emptyset$ has a Hamel basis **(6 x 2 = 12)**

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. (i) Prove that in a finite dimensional Normed space X, any subset $M \subset X$ is compact if and only if M is closed and bounded

    (ii) State and prove Riesz's lemma.
20. (i) Let T be a linear operator. Then prove that (a) the range $R(T)$ is a vector space; (b) if $\dim D(T) = n < \infty$, then $\dim R(T) \leq n$; (c) The null space $N(T)$ is a vector space.

    (ii) Let T be a bounded linear operator. Then prove that (a) $x_n \rightarrow x$ (where $x_n, x \in D(T)$) implies $Tx_n \rightarrow Tx$. (b) The null space $N(T)$ is closed.
21. (i) State and prove Bessel inequality.

    (ii) Explain Gram-Schmidt process
22. State and prove Hahn-Banach Theorem of extension of linear functionals.

(2 x 5 = 10)

QP Code **Reg. No. …………..**

Name ………………

M Sc (Mathematics)Degree (C.S.S) Examination, ………

Fourth Semester

Faculty of Science

ME010401 SPECTRAL THEORY

(2019 admissions onwards)

Time: Three hours Max. Weight: 30

Section- A

(Answer any **eight** questions. Each question carries a weight of 1)

1. Let X and Y be normed spaces, $T \in B(X,Y)$ and (x_n) a sequence in X. If $x_n \overset{w}{\rightarrow} x$ then show that $Tx_n \overset{w}{\rightarrow} Tx$
2. Of what category is the set of all rational numbers (a) in $\mathbb{R}$ (b) in itself. Of what category is the set of all integers (c) in $\mathbb{R}$ (d) in itself?
3. Prove that all matrices representing a given linear operator $T: X \rightarrow X$ on a finite dimensional Normed space X relative to various bases for X have the same eigen values.
4. Show that the spectrum of a bounded linear operator on a complex Banach space is bounded.
5. Let $S, T \in B(X,X)$, show that for every $\lambda \in \rho(S) \cap \rho(T)$, $R_\lambda(S) - R_\lambda(T) = R_\lambda(S)(T-S)R_\lambda(T)$
6. Define Compact linear operator. Prove that the identity operator on an infinite dimensional space is not compact.
7. Prove that the operator $T: l^2 \rightarrow l^2$ defined by $y = (\eta_j) = Tx$ where $\eta_j = \frac{\zeta_j}{j}$ for $j = 1,2, \dots$ is compact
8. Prove that every non zero spectral value of a compact linear operator is an eigen value.
9. Let $T: H \rightarrow H$ be a bounded self-adjoint linear operator on a complex Hilbert space. Then prove that all eigen vectors corresponding to different eigen values of T are orthogonal.
10. If P_1 and P_2 be projections on a Hilbert space H. Then prove that the difference $P_1 - P_2$ is a projection on H if and only if $Y_1 \subset Y_2$ where $Y_j = P_j(H)$

(8 x 1 = 8)

Section B

(Answer any **six** questions. Each question carries a weight of 2)

11. Prove that every Hilbert space is reflexive.
12. (a) Prove that Strong convergence implies weak convergence with the same limit.
    (b) Is the converse true? Justify
13. State and prove Banach fixed point theorem.
14. Show that the spectrum of a bounded linear operator on a complex Banach space is non-empty.
15. If (x_n) and (y_n) are Cauchy sequences in a Normed algebra A, show that
    (a) $(x_n y_n)$ is Cauchy in A
    (b) If $x_n \to x$ and $y_n \to y$ then $x_n y_n \to xy$
16. Prove that a compact linear operator $T: X \to Y$ from a Normed space X into a Banach space Y has a compact linear extension $\tilde{T}: \hat{X} \to Y$ where $\hat{X}$ is the completion of X
17. Let H be a complex Hilbert space and $T: H \to H$ be a bounded self-adjoint linear operator. Then prove that $m = \inf_{||x||=1} \langle Tx, x \rangle$ and $M = \sup_{||x||=1} \langle Tx, x \rangle$ are spectral values of T.
18. Prove that a bounded linear operator $P: H \to H$ on a Hilbert space H is a projection if and only if P is self adjoint and idempotent (**6 x 2 = 12**)

Section C

(Answer any **two** questions. Each question carries a weight of 5.)

19. State and prove Open Mapping Theorem.
20. (i) State and prove spectral mapping theorem for polynomials
    (ii) Show that for any operator $T \in B(X, X)$ on a complex Banach space X, $r_\sigma(\alpha T) = |\alpha|\, r_\sigma(T)$ and $r_\sigma(T^k) = [r_\sigma(T)]^k, k \in \mathbb{N}$
21. (i) Prove that the set of eigen values of a compact linear operator $T: X \to X$ on a normed space X is countable and the only possible point of accumulation is $\lambda = 0$
    (ii) Let $T: X \to X$ be a compact linear operator and $S: X \to X$ a bounded linear operator on a Normed space X. Then prove that ST and TS are compact
22. (i) Let $T: H \to H$ be a bounded self adjoint linear operator on a complex Hilbert space H. Prove that a number λ belongs to the resolvent ser $\rho(T)$ if and only if there exists a $c > 0$ such that for every $x \in H$, $\|T_\lambda x\| \geq c\|x\|$
    (ii) Show that the spectrum $\sigma(T)$ of a bounded linear operator $T: H \to H$ on a complex Hilbert space H is real. . (**2 x 5 = 10)**