

**BSc. Mathematics Degree (MGU-CBCSS-UG) Examination
(Model Question)**

MM2CRT02 II Semester BSc Mathematics (Core) Model I

2017 admissions onwards

Part A Each Question Carries 2 marks. Answer Any 10 Questions

1. Find the condition that the line $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$
2. Define Director circle and auxiliary circle of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
3. What is the equation of the chord joining two points t_1 and t_2 of the parabola $y^2 = 4ax$
4. Find the polar of the point (x_1, y_1) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
5. Derive the polar equation of a circle with pole at the circumference and diameter as the initial line
6. Find the eccentricity and the semi latus rectum of the conic $\frac{10}{r} = 3\cos\theta + 4\sin\theta + 5$
7. Define the Hyperbolic functions $\sinh x$ and $\cosh x$
8. Show that $\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$
9. Prove that $\log(-1) = i\pi$
10. If $x = \log s$ and $y = s^2 - 1$ find $\frac{d^2y}{dx^2}$
11. If $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$
12. Determine $\lim_{x \rightarrow 0} \frac{\sinh x - x}{\sin x - x \cos x}$

Part B Each Question Carries 5 marks. Answer Any 6 Questions

13. Find the equation of the tangents drawn to the ellipse $9x^2 + 16y^2 = 144$ from the point (2,3)
14. Find the equation of the chord joining any two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
15. Find the equation of a tangent at any point on the circle $r = 2a \cos \theta$
16. If $\tan \frac{\theta}{2} = \tanh \frac{u}{2}$ prove that $u = \log \tan \left[\frac{\pi}{4} + \frac{\theta}{2} \right]$
17. Sum the series $\cos \alpha \cos \alpha + \cos^2 \alpha + \cos^3 \alpha \cos 3\alpha + \dots$
18. Resolve into real factors $x^{10} + 1$
19. Find the n^{th} derivative of $\frac{x+1}{6x^2-7x+3}$
20. If $y = [\log(x + \sqrt{1+x^2})]^2$, prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} - n^2y_n = 0$
21. Determine $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$

Part C Each Question Carries 15 marks. Answer Any 2 Questions

22. A. A common tangent is drawn to the circle $x^2 + y^2 = c^2$ and the parabola $y^2 = 4ax$. Show that the angle θ which makes with the x axis is given by $\tan^2 \theta = \frac{-c + \sqrt{c^2 + 4a^2}}{2c}$
B. Find the equation of the tangents drawn to the ellipse $9x^2 + 16y^2 = 144$ from the point (2,3)

23. A. Show that the semi latus rectum is the harmonic mean between the segments of any focal chord
B. Prove that the tangents at the extremities of any focal chord of a conic intersect on the corresponding directrix
24. A. Resolve into real factors $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$
B. Sum the series $1 - \frac{1}{2} \cos \theta + \frac{1.3}{2.4} \cos 2\theta - \frac{1.3.5}{2.4.6} \cos 3\theta + \dots$
25. A. If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$
B. Find $\lim_{x \rightarrow 1} ((1 - x^2))^{\frac{1}{\log(1-x)}}$