Model Question Paper

Fourth Semester M.Sc. Degree Examination (CSS)

ST4E03: STATISTICAL RELIABILITY ANALYSIS

Time : 3 hours **Total weights: 30**

Part A

(**Answer any 5 questions. Weightage 1 for each question**)

1. Define series system and parallel system with *n* components. Write down the structure functions of series system and parallel system.
2. What is meant by reliability of a system? Obtain the reliability of a parallel system by stating your assumptions.
3. Discuss the role of exponential distribution in reliability study.
4. Define hazard rate and find the hazard rate of Weibull distribution.
5. Define the concept of DFR distributions. Is exponential distribution IFR or DFR?
6. Explain IFRA and DMRL ageing properties.
7. Let $X_1, X_2, ..., X_n$ be a random sample from a population having distribution function F. Obtain a non-parametric estimate of the reliability function.
8. Explain Type I censoring in reliability.

Part B

(**Answer any 5 questions. Weightage 2 for each question**)

9. Define a coherent structure and show that the dual of a coherent structure is coherent.

10. If $T_1, T_2, ..., T_n$ are associated random variables, then show that $P\left(\min_{1\le i\le n} T_i > t\right) \ge \prod_{i=1}^{n} P(T_i > t)$ and $P\left(\max_{1\le i\le n} T_i > t\right) \le \coprod_{i=1}^{n} P(T_i > t)$

11. Let $h(t) = \begin{cases} \alpha & \text{for } 0 < t < \tau \\ \beta & \text{for } \tau \le t < \infty \end{cases}$, for some $\tau > 0$, be the hazard rate of a random variable. Derive its reliability function.

12. What is a proportional hazard model? Show that the exponential and weibull models are special cases of it.

13. If *F* is *IFR* and *F*(*z*) < 1, then show that *F* has a probability density on $(-\infty, z)$.

14. Show that *F* is *DFRA* if and only if $\bar{F}(\alpha t) \le \left[\bar{F}(t)\right]^{\alpha}$, $t \ge 0, 0 < \alpha < 1$.

15. Obtain the MLE for the reliability function based on Type-2 censored samples from an exponential distribution.

16. Find the Kaplan-Meier estimate for the following data:
3, 5, 5+, 6, 7, 8, 9, 10, 10+, 11 (Here + denotes the censored life time).

Part C

(**Answer any 3 questions. Weightage 5 for each question**)

17. Let φ be a coherent structure of independent components. Then show that

$$\prod_{j=1}^{k} \coprod_{i \in K_j} p_i \le P[\varphi(\underline{x}) = 1] \le \coprod_{j=1}^{p} \prod_{i \in P_j} p_i ,$$

where $K_1, K_2, ..., K_k$ are the minimal cut sets and $P_1, P_2, ..., P_p$ are the minimal path sets corresponding to φ.

18. (i) Explain the terms (a) Reliability function and (b) mean residual life function.
(ii) Let $m(t)$ be the mean residual life function and $R(t)$ the survival function of a continuous lifetime random variable T. Prove that:
(a) $$m(t) = \frac{1}{R(t)} \int_t^{\infty} R(x)dx$$
(b) $$R(t) = \frac{m(0)}{m(t)} \exp\left[-\int_0^t \frac{1}{m(u)} du \right].$$

19. Prove the following implications:
(i) $IFR \Rightarrow IFRA \Rightarrow NBU \Rightarrow NBUE$.
(ii) $IFR \Rightarrow DMRL \Rightarrow NBUE$.

20. Prove that *IFR* class is closed under the formation of convolutions. What about the closure of *DFR* class under convolutions?

21. Derive Kaplan-Meier estimate of the reliability function and state its properties.

22. Derive the stress-strength reliability when strength (*S*) and load (*L*) follow Exponential distribution.