Model Question Paper

Second Semester M.Sc Degree Examination (CSS)

ST2C09 -STOCHASTIC PROCESS

Time: 3 hours Total Weights: 30

Part A

(***Answer any five questions. Weightage 1 for each question.***)

1. Define a counting process and give an example.
2. What do you mean by a covariance stationary process?
3. If the offspring distribution is $\left(\frac{1}{3}, 0, \frac{2}{3}, 0, \ldots, 0\right)$, derive the probability π of ultimate extinction of the branching process.
4. Define an ergodic Markov chain and give one example.
5. Write down the n step transition probabilities of a symmetric random walk on integers in one dimension.
6. Describe delayed renewal process.
7. Define infinitesimal generator of a continuous parameter Markov chain.
8. Obtain the renewal function if the inter renewal times have density:

$$f(x) = \lambda^2 x \mathrm{e}^{-\lambda x};\ x > 0,\ \lambda > 0.$$

Part B

(***Answer any* five *questions. Weightage* 2 *for each question.***)

9. Show that a Markov chain is completely determined by the one-step transition probability matrix and initial probability vector.
10. Show that in an irreducible Markov chain, all states are either recurrent or all are transient.
11. State and prove elementary renewal theorem.
12. Let μ be the expected number of offsprings in each generation in a Galton - Watson branching process. Show that if $\mu \leq 1$, the process dies out with probability one.
13. If $X_1(t)$ and $X_2(t),\ t \geq 0$ are two independent Poisson process with intensity parameters λ_1 and λ_2 respectively, find the distribution of $X_1(t)$ given that $X_1(t) + X_2(t) = n$

14. Define stopping time. If $N(t)$ is a renewal process associated with the i.i.d sequence $\{X_n\}$ show that $N(t)+1$ is a stopping time for $\{X_n\}$.

15. Show that for branching process $\phi_{n+1}(s) = \phi_n(\phi(s))$ under usual notation.

16. Prove that if the intervals between successive occurrences of an event E are independently distributed with a common exponential distribution with mean $\frac{1}{\lambda}$, then the events E form a Poisson process with mean λt.

Part C

(***Answer any*** **three** ***questions. Weightage*** **5** ***for each question.***)

17. Obtain the ultimate ruin probabilities of a classical Gambler's ruin problem.

18. Derive the birth and death process from a set of postulates to be stated. Obtain the difference differential equation.

19. State and prove the mean ergodic theorem.

20. Obtain the ultimate extinction probabilities of a Galton-Watson branching process.

21. Discuss the steady state behaviour of a M/M/1 queue. Derive the waiting time distributions associated with this model.

22. (a) Show that the probability of absorption into recurrent class from a transient class is unity.

    (b) In a finite Markov chain, show that not all states are transient or null recurrent.