Model Question Paper

Second Semester M.Sc Degree Examination (CSS)

ST2C08 -STATISTICAL ESTIMATION THEORY

Time: 3 hours   Total Weights: 30

Part A

(***Answer any five questions. Weightage 1 for each question.***)

1. Find an unbiased estimator of $2\theta^2 - 7\theta + 5$, where θ is the binomial parameter in $f(x, \theta) = \theta^x(1-\theta)^{1-x}, 0 < \theta < 1$ and $x = 0$ or 1.

2. Obtain the Cramer-Rao lower bound for the variance of an unbiased estimator of θ in sampling from N $(\theta,\ 1)$.

3. Define completeness. Show that the family with density :

$$\begin{aligned} f_\theta(x) &= \frac{nx^{n-1}}{\theta^n}, \text{for } x > 0, \theta > 0 \\ &= 0, \text{otherwise} \end{aligned}$$

   is complete.

4. Define an UMVU estimator and give an example.

5. Explain Fisher's scoring method of estimation.

6. Let F(.) be the distribution function of N $(\mu,\ \sigma^2)$, where μ and σ^2 are unknown. Find the MLE of F(t) for a given t.

7. Explain the basic problem of decision theory with its important elements.

8. What is meant by minimax estimator ? Indicate a method of determining minimax estimates.

Part B

(***Answer any* five *questions. Weightage 2 for each question.***)

9. Show that if $s_1 = \frac{1}{n}\sum|X_i - \mu|$ is the mean deviation, where $X_1, X_2, \ldots, X_n$ is a random sample of size n from N(μ, σ^2), then $\sqrt{\frac{\pi}{2}}\ s_1$ is an unbiased estimator of σ with efficiency $\frac{1}{\pi-2}$. Is it also consistent.

10. What is Minimum Variance Bound (MVB) estimator ? Obtain the conditions under which MVB estimator exists. Explain with a suitable example.

11. State and prove Basu's theorem.

12. Let $X_1,\ X_2, \ldots X_n$ be independent observations from a Poisson distribution with parameter θ. Define $\phi(X_1) = \begin{cases} 1 & \text{if} \quad X_1 = 0 \\ 0 & \text{if} \quad X_1 > 0. \end{cases}$

    (i) Show that $\phi(X_1)$ is unbiased for $e^{-\theta}$.

    (ii) Obtain minimum variance unbiased estimator of $e^{-\theta}$ and compute its variance.

13. Explain the minimum Chi-square and modified minimum Chi-square methods of estimation.

14. Let X_1 and X_2 be two independent observations from a distribution with p.d.f

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}, & x > \theta \\ 0 & \text{otherwise} \end{cases}$$

    and let $Y = min(X_1, X_2)$. Find the confidence coefficient of the interval $\left[Y - \frac{1}{2}, Y + \frac{1}{2}\right]$.

15. Explain the terms (i) Loss function (ii) Risk function (iii) Baye's estimate (iv) Minimax estimation.

16. Show that estimation and testing can be viewed as particular cases of a decision problem.

Part C

(*Answer any* three *questions. Weightage* 5 *for each question.*)

17. Define Bhattacharyya's bounds for the variance of an unbiased estimator of θ. Show that it forms an increasing sequence.

18. State and prove Fisher-Neymann Factorization theorem.

19. State and prove Rao-Blackwell theorem.

20. Define M.L.E. of a parameter and prove that M.L.E's are consistent.

21. Let $\Theta = (0, \infty), R =$ real line and $L(\theta, a) = (\theta - a)^2$. Let X follows Poisson with parameter $\theta > 0$ and the prior distribution of θ be the Gamma (α, β) with density:

$$g(\theta) = (\Gamma(\alpha)\beta^{\alpha})^{-1} e^{-\theta/\beta}\ \theta^{\alpha-1} \text{ for } \theta > 0 (\alpha > 0, \beta > 0).$$

    Show that the posterior distribution of θ given $X = x$ is the gamma distribution $G\left[\alpha + x, \frac{\beta}{\beta+1}\right]$.

22. Let X have a binomial distribution with parameters $(n, \theta), 0 < \theta < 1$. Using the loss function

$$L(\theta, \alpha) = \frac{(\theta - \alpha)^2}{\theta(1-\theta)},$$

    Show that $d(X) = \frac{X}{n}$ is a minimax estimate of θ with constant risk $\frac{1}{n}$.