Model Question Paper

Second Semester M.Sc Degree Examination (CSS)

ST2C07 -ADVANCED PROBABILITY THEORY

Time: 3 hours | Total Weights: 30

Part A

(*Answer any five questions. Weightage 1 for each question.*)

1. Define signed measure and show that difference of two measures is a signed measure.
2. Define absolutely continuous and singular measures and show that two measures are continuous and singular if and only if they are zero measures.
3. Define martingale and give an example.
4. Show that $E\{E(X|\mathfrak{D})\} = E(X)$, whenever $E|X| < \infty$.
5. Obtain the characteristic function of a random variable with density $f(x) = \begin{cases} 1-|x|; & |x| < 1 \\ 0; & |x| > 1. \end{cases}$
6. If $\phi(t)$ is a characteristic function, show that $Re(\phi(t)))$ is a characteristic function.
7. State Lindberg-Feller Central limit theorem.
8. Define conditional expectation of a random variable given a σ-filed.

Part B

(*Answer any* five *questions. Weightage* 2 *for each question.*)

9. State and prove Khinchin's form of WLLN.
10. State and prove a sufficient condition for holding SLLN.
11. Let $\Omega = (0,1)$ and $\mathfrak{B}$ be the Borel field of subsets of Ω and P be the Lebesgue measure on $(\Omega, \mathfrak{B})$. Let $\mathfrak{D}$ be the minimal σ-field containing the class of sets $\{(0, \frac{1}{8}], (\frac{1}{8}, \frac{5}{8}), [\frac{5}{8}, 1)\}$. Define X on Ω as $X(\omega) = \omega^2$. Find $E(X|\mathfrak{D})$.
12. Show that $\{X_n\}_{n=1}^{\infty}$ is a martingale if and only if $E(X_n|X_1, \ldots, X_m) = X_m$ a.s., $\forall\ m < n$.
13. If X is a $\mathfrak{B}$ measurable function, then show that $E(XY|\mathfrak{B}) = XE(Y|\mathfrak{B})$.
14. Obtain the characteristic function of a random variable following the standard Cauchy distribution.
15. Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of independent random variables such that $X_n \sim N(n, n^2)$. Check whether the sequence obeys CLT.

16. Examine if SLLN holds for the sequence $\{X_n\}$ of independent random variables where $P(X_n = \pm\, n^{\alpha}) = \frac{1}{2n^2}, P(X_n = 0) = 1 - \frac{1}{n^2}$.

Part C

(***Answer any* three *questions. Weightage* 5 *for each question.***)

17. State and prove Lindberg-Levy form of Central limit theorem.
18. State and prove Lebesgue decomposition theorem.
19. State and prove Fubini's theorem.
20. State and prove a necessary and sufficient condition for a sequence of i.i.d. random variables to obey WLLN.
21. State and prove inversion theorem.
22. State and prove Lindberg-Levy form of Central limit theorem.