Model Question Paper

Second Semester M.Sc Degree Examination (CSS)

ST2C06 -MULTIVARIATE DISTRIBUTIONS

Time: 3 hours Total Weights: 30

Part A

(***Answer any five questions. Weightage 1 for each question.***)

1. Suppose (X, Y)$\sim$ BN $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$. Find E(X|Y).
2. Define Gumbel bivariate exponential distribution. State any *one* property of the same.
3. Define Singular multivariate normal distribution.
4. Define Multinomial distribution. Give any *two* applications of the same.
5. Find the variance and covariance of normal variates which have the quadratic form

$$3y_1^2 + 2y_2^2 + 2y_3^2 + 4y_1y_2 + 2y_2y_3$$

   in their distribution.

6. Let X$\sim$N$_p(\mu, \Sigma)$. Write down the characteristic function of X.
7. Define Wishart distribution.
8. Define Multiple correlation and Partial correlation coefficients.

Part B

(***Answer any* five *questions. Weightage 2 for each question.***)

9. Define the bivariate Marshall-Olkin exponential distribution and point out its applications. Derive its m.g.f.
10. Explain a multinomial distribution. Show that $Corr(X_i, X_j) = -\sqrt{\frac{p_ip_j}{(1-p_i)(1-p_j)}}$ under usual notations.
11. If $X = \begin{bmatrix} X^{(1)} \\ -- \\ X^{(2)} \end{bmatrix}$ follows $N_p(\mu, \Sigma)$, find the conditional distribution of $X^{(1)}$ on $X^{(2)}$. Show that the regression of $X^{(1)}$ on $X^{(2)}$ is linear.
12. If $Y = AXA'$, obtain the Jacobian of the transformation when X is (i) symmetric (ii) not symmetric.
13. Derive the null distributions of the sample correlation coefficient and sample partial correlation coefficient.

14. If $X = TT'$ where T is a lower triangular matrix, find the Jacobian of the transformation.

15. The exponent of a bivariate normal density function is

$$-\frac{1}{24}(x^2 - 2xy + 4y^2 - 10x - 20y + 100).$$

Determine all the parameters of the distribution.

16. Suppose (X, Y)$\sim$ BN $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$. Show that if $\rho = 0$, then X and Y are independent. Is the converse true?

Part C

(*Answer any* three *questions. Weightage* 5 *for each question.*)

17. State and prove Cochran's theorem on quadratic forms.

18. If $X \sim N_p(0, \Sigma)$, show that the quadratic form $X'AX$ follows Chi-square distribution with r d.f. iff $A\Sigma$ is idempotent of rank r.

19. (a) State and establish the reproductive property of Wishart distribution $W_p(n, \Sigma)$.

    (b) Show that any principal submatrix of a Wishart matrix is also a Wishart matrix.

20. Derive the M.L.E's of μ and Σ in $N_p(\mu, \Sigma)$ and show that they are independently distributed.

21. Show that $X \sim N_p(\mu, \Sigma)$ if and only if X can be written in the form $X = \mu + BY$, under conditions to be stated.

22. Derive the characteristic function of a Wishart matrix and hence derive its distribution.