Model Question Paper

First Semester M.Sc Degree Examination (CSS)

ST1C04 - MATHEMATICAL METHODS FOR STATISTICS

Time: 3 hours Total Weights: 30

Part A

(***Answer any five questions. Weightage 1 for each question.***)

1. If $L\{F(t)\} = f(s)$, then find $L\{F(at)\}$.
2. Define Riemann integral of a function.
3. Define a monotone class and a σ-field. Establish the relation between them.
4. Briefly explain a Lebesgue-Stieltjes measure.
5. Give an example of a function which is Lebesgue integrable but not Riemann integrable.
6. Let f be a non-negative measurable function. Show that $\int f d\mu = 0$ implies f=0 a.e.
7. When do you say that a function is analytic in a domain D.
8. Write down the Cauchy's integral formula.

Part B

(***Answer any* five *questions. Weightage* 2 *for each question.***)

9. Test for convergence of the infinite series $\sum_{1}^{\infty} \frac{n+1}{n^p}$.
10. Explain the method of Lagrangian multipliers.
11. Define a field and give an example. Show that the intersection of arbitrary number of fields is a field and union of arbitrary number of fields need not be a field.
12. If f and g are two measurable real-valued functions defined on the same domain, then show that the functions $f + g$ and $f - g$ are also measurable.
13. For a finite measure μ, show that $\mu(\lim A_n) = \lim \mu(A_n)$.
14. Give the general definition of integral of a measurable function and state its elementary properties.
15. State and prove a sufficient condition for function f(z) to be analytic.
16. Find the residues of $\frac{z+1}{z^2(z-2)}$ at its poles.

Part C

(***Answer any* three *questions. Weightage* 5 *for each question.***)

17. (i) State and prove a necessary and sufficient condition for the existence of Riemann integral.

    (ii) Find maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

18. (i) If L denotes the Laplace transform and $L[F(T)] = f(s)$, show that

$$L\left[\frac{dF(t)}{dt}\right] = sf(s) - F(0).$$

    (ii) Using Laplace transform solve $y'' + 2y' + y = 6te^{-1}$, given that $y(0) = 2, y'(0) = 5$.

19. State and prove Lebesgue Dominated Convergence Theorem.

20. What is a simple measurable function? Show that a non-negative measurable function can be viewed as the pointwise limit of a non-decreasing sequence on non-negative simple functions.

21. State and prove Fatou's lemma.

22. (i) State and prove Liouville's theorem.

    (ii) Show that $\displaystyle\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} = \frac{2\pi}{\sqrt{3}}$.