Model Question Paper

First Semester M.Sc Degree Examination (CSS)

ST1C03 - PROBABILITY THEORY

Time: 3 hours | Total Weights: 30

Part A

(***Answer any five questions. Weightage 1 for each question.***)

1. Distinguish between a field and a σ-field. Let $\Omega = \{1, 2, 3, \ldots\}$ and C be the class of subsets A of Ω such that either A contains a finite number of points or A^c contains a finite number of points. Examine whether C is field or a σ-field or both.
2. Define convergence in quadratic mean. Give an example.
3. Show that for a sequence of random variables $\overline{\lim}\ X_n$ and $\underline{\lim}\ X_n$ are random variables.
4. Explain the concept of independence of a collection of random variables.
5. Prove that uncorrelated random variables need not be independent.
6. Define convergence in probability and convergence almost surely and establish the implication between them.
7. Show that $E|X|^r < \infty$ implies $E|X|^s < \infty$ for $s < r$.
8. Show that $E|Y|$ is finite if and only if $\sum_{n=1}^{\infty} P\{|Y| > n\} < \infty$.

Part B

(***Answer any* five *questions. Weightage* 2 *for each question.***)

9. State and prove the continuity property of probability measure.
10. A non-symmetric coin is tossed until a head appears. Give a space Ω, a σ-field F and a probability measure P on F to describe the probability space associated with the experiment.
11. If X and Y are independent random variables, show that $E(XY) = E(X).E(Y)$, whenever both sides exist.
12. Show that every σ-field is a monotone field. Is the converse true? Justify.
13. Define distribution functions of a random variable. State its important properties. Show that a distribution function can have at most a countable number of discontinuities.

14. State and establish Markov inequality. Deduce Tchebysev's inequality.

15. Define convergence is probability and convergence in distribution regarding a sequence of random variables $\{X_n,\ n \geq 1\}$ to a random variable X. Show that the former implies the latter.

16. Prove or disprove: Convergence almost surely implies convergence rth mean.

Part C

(*Answer any* three *questions. Weightage* 5 *for each question.*)

17. State and prove Borel zero-one law.

18. Define the Distribution Function of a random variable. State and prove the decomposition theorem on distribution functions.

19. State and prove Baye's theorem.

20. (a) Explain the various modes of convergence of sequence of random variables and give their mutual implications.

    (b) If $\{X_n\}$ is independent, then $X_n \xrightarrow{a,s} 0$ if and only if $\sum P[|X_k| > c] < \infty, c > 0$.

21. Show that if $A_n = P\{|X| \geq n\}, n = 0, 1, 2, \ldots, \sum_{n=1}^{\infty} P(A_n) \leq E(|X|) \leq 1 + \sum_{n=1}^{\infty} P(A_n)$.

22. State and prove Kolmogorov's inequality.