Model Question Paper

First Semester M.Sc Degree Examination (CSS)

ST1 C02: ANALYTICAL TOOLS FOR STATISTICS

Time: 3 hours | Total Weights: 30

Part A

(*Answer any five questions. Weightage 1 for each question.*)

1. Define (i) Vector space and (ii) Basis of a vector space.
2. Give two definitions of rank of a matrix.
3. Define algebraic multiplicity and geometric multiplicity of characteristic roots.
4. Show that two similar matrices have the same characteristic roots.
5. Write down the symmetric matrix associated with the quadratic form $x^2 - xy + 3y^2$.
6. Define (i) Index of a quadratic form and (ii) Signature of a quadratic form.
7. Define a convex set.
8. What are slack and surplus variables.

Part B

(*Answer any* five *questions. Weightage* 2 *for each question.*)

9. Determine whether the vectors (1, 2, 6), (-1, 3, 4), (-1, -4, -2) are linearly independent.
10. Explain the method of computing the inverse of a matrix by partition.
11. State and Prove Cayley-Hamilton Theorem.
12. Show that every matrix has a g-inverse. If $\bar{A}$ is a g-inverse of A, then show that $\bar{A}A$ is idempotent.
13. State and prove a necessary and sufficient condition for positive definiteness of a quadratic form.
14. Explain the spectral decomposition of a real symmetric matrix and state its properties.
15. Show that the set of feasible solutions to an LPP is a convex set.
16. Define the primal and dual of an LPP. If the primal has a finite optimum solution, then show that the dual also will have a finite optimum solution.

Part C

(***Answer any*** **three** ***questions. Weightage*** **5** ***for each question.***)

17. Explain Gram-Schmidt method of constructing an orthonormal basis of a finite dimensional vector space. Illustrate it using an example.

18. Determine a basis for the null space of

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{bmatrix}.$$

19. Define Moore-Penrose g-inverse. Determine the Moore-Penrose g-inverse of

$$A = \begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & -2 \end{bmatrix}.$$

20. Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal with diagonal elements as the characteristic roots of A when

$$A = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}.$$

21. Examine the definiteness of the quadratic form $6x^2 + 3y^2 + 14z^2 + 4y + 18xz + 4xy$ after reducing it to its canonical form.

22. Use simplex method to solve the LPP:

    Maximize $Z = x_1 - 3x_2 + 2x_3$

    Subject to

$$3x_1 - x_2 + 2x_3 \leq 7, \quad -2x_1 + 4x_2 \leq 12, \quad -4x_1 + 3x_2 + 8x_3 \leq 10,$$

$$x_1, x_2, x_3 \geq 0$$