Model Question Paper

First Semester M.Sc Degree Examination (CSS)

ST1CO1 - DISTRIBUTION THEORY

Time: 3 hours | Total Weights: 30

Part A

(***Answer any five questions. Weightage 1 for each question.***)

1. Define logistic distribution.
2. Obtain the probability generating function of negative binomial random variable. Hence find its variance.
3. Show that the exponential distribution possesses the 'lack of memory property'.
4. Define Pareto Distribution and mention its important characteristics.
5. Define a truncated distribution. Write the p.m.f. of a truncated Poisson distribution.
6. If X has a Beta (m, n) distribution, then show that $Y = \frac{n}{m}\frac{X}{1-X}$ has $F(2m, 2n)$ distribution.
7. Define a t-statistic. Mention its applications.
8. If $X_1, X_2, \ldots, X_n$ are iid random variables having exponential distribution with pdf

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x > 0,\ \theta > 0 \\ 0, & \text{otherwise} \end{cases}.$$

Obtain the distribution of $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$.

Part B

(***Answer any* five *questions. Weightage 2 for each question.***)

9. Define hypergeometric distribution. Find its mean and variance.
10. Obtain Poisson distribution as a limiting case of negative binomial distribution.
11. Describe log normal distribution. Obtain its moment generating function and determine its coefficient of variation.
12. If X has Cauchy distribution with p.d.f. $f(x) = \frac{1}{\pi(1+x^2)}$ $-\infty < x < \infty$, show that the distribution of $\frac{1}{X}$ is again Cauchy. Is the converse true? Establish your claim.
13. If X and Y are independent Rectangular variates on [0, 1], find the distribution of $\frac{X}{Y}$.

14. If X and Y are independent Gamma variates with parameters μ and γ respectively, find the distribution of $\frac{X}{X+Y}$ and hence obtain $E\left[\frac{X}{X+Y}\right]$.

15. Find the p.d.f. of the sample median for a sample of size 7 drawn from a population having p.d.f.,

$$f(x) = \begin{cases} e^{-x} & 0 \leq x < \infty \\ 0, & \text{elsewhere.} \end{cases}$$

16. Derive the joint distribution of X(r) and X(s) the r^{th} and s^{th} order statistics.

Part C

(*Answer any* three *questions. Weightage* 5 *for each question.*)

17. Define generalised power series distribution. Obtain its m.g.f. Show that the Binomial distribution can be considered as a particular case of the g.p.s.d. Hence obtain its mean and variance.

18. Find the moment generating function of the normal distribution $N(\mu,\ \sigma^2)$ and deduce that $\mu_{2n+1} = 0,\ \mu_{2n} = 1.3.5\ldots(2n-1)\sigma^{2n}$, where μ_n denotes the n^{th} central moment.

19. Explain Weibull Distribution. Derive its Hazard Rate function. Show that $\min(X_1, X_2, \ldots, X_n)$ follows Weibull if and only if X_i's follows Weibull Distribution.

20. Let $X \sim N(0,1)$ and $Y \sim N(0,1)$ be independent random variables. Obtain the distribution of $\frac{|X|}{|Y|}$.

21. Let $X \sim \beta_1(\mu,\nu)$ and $Y \sim \Gamma(\lambda, \mu+\nu)$ be independent random variables, $(\mu, \nu, \lambda > 0)$. Find the pdf of XY and identify its distribution.

22. Let $X_1, \ldots, X_n$ be a random sample from $U[0,1]$. Derive the distribution of the midrange.