B.TECH. DEGREE EXAMINATION, MAY 2011

<u>MODEL QUESTION PAPER</u>

First and Second Semester

ENGINEERING MATHEMATICS – I

(Common to all branches)

Time : Three Hours　　　　　　　　　　　　　　　　　　　　Maximum : 100 Marks

PART A

1. Define eigen values and eigen vectors of a matrix. Find the sum of the eigen values of

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

2. State Euler's theorem on homogeneous functions. If $\tan u = \frac{x^3 + y^3}{x - y}$, prove that $x\frac{du}{dx} + y\frac{du}{dy} = \sin 2u$

3. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \;\; x + y \;\; dxdy$

4. Solve $\;\; D^2 + 1 \;\; y = \cos(2x - 1)$

5. State the first shifting property in Laplace transforms. Also find $L \;\; e^{-3t} t^3 \;\;$.

[5 x 3 marks = 15 marks]

PART B

6. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -1 & -4 \\ 3 & 1 & 3 & -2 \end{bmatrix}$

7. If $u = f(x, y)$ where $x = r\cos\theta$ *and* $y = r\sin\theta$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.

8. Evaluate the integral $\int_0^4 \int_x^4 \frac{x}{x^2 + y^2} dxdy$ by changing the order of integration.

9. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

10. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.

[5 x 5 marks = 25 marks]

Turn over

PART C

Module I

11. (a) Find values of a and b for which the equations $x + ay + z = 3$, $x + 2y + 2z = b$, $x + 5y + 3z = 9$ are consistent.

(7 marks)

(b) Show that the vectors (2, −2, 1), (1, 4, −1) and (4, 6, −3) are linearly independent.

(5 marks)

Or

12. Reduce the quadratic from $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to a canonical form by orthogonal transformation. and specify the matrix of transformation. Also find the rank, index, signature and nature of the quadratic form.

(12 marks)

Module II

13. (a) If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, −1, 0). (5 marks)

(b) Expand $x^2y + 3y - z$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem. (7 marks)

Or

14. (a) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ (7 marks)

(b) In a plane triangle, find the maximum value of *cosA cosB cosC*. (5 marks)

Module III

15. (a) Find, by triple integration, the volume of the sphere $x^2 + y^2 + z^2 = a^2$ (5 marks)

(b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 \frac{dzdydx}{\sqrt{x^2+y^2+z^2}}$ (7 marks)

Or

16. (a) Find the area between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ lying in the first quadrant, by double integration.

(5 marks)

(b) By transforming into cylindrical coordinates, evaluate the integral $\iiint x^2 + y^2 + z^2 \; dx\,dy\,dz$ taken over the region of space defined by $x^2 + y^2 \le 1$ and $0 \le z \le 1$.

(7 marks)

Module IV

17. (a) Solve $D^2 - 4D + 3 \; y = \sin 3x \cos 2x$ (7 marks)

(b) Solve $x\frac{d^2y}{dx^2} - 2\frac{y}{x} = x + \frac{1}{x^2}$ (5 marks)

Or

18. (a) Solve $D - 2 \;^2 y = 8 \; e^{2x} + \sin 2x + x^2$ (7 marks)

(b) Solve $1 - x \;^2 \frac{d^2y}{dx^2} + \; 1 + x \; \frac{dy}{dx} + y = 2\sin\left[\log \; 1 + x \;\right]$ (5 marks)

Module V

19. (a) Find the inverse Laplace transform of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$ (5 marks)

(b) Using convolution theorem, find the inverse Laplace transform of $\frac{s}{s^2 + a^2 \;^2}$ (7 marks)

Or

20. Solve the following differential equation by the method of Laplace transforms

$y'' - 3y' + 2y = 4t + e^{3t}$, *when* $y(0) = 1$ *and* $y'(0) = -1$

(12 marks)

[5 x 12 marks = 60 marks]