

MAHATMA GANDHI UNIVERSITY
M.C.A DEGREE EXAMINATION
MODEL QUESTION PAPER
(2011 Revised Syllabi)
First Semester

MCA101 Mathematical Foundations of Computer Science

Time: 3 hours

Maximum: 75Marks

Part A

*Answer any ten questions.
Each question carries 3 marks.*

1. If A, B, C are sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$ then show that $B = C$
2. Prove that the relation "congruence modulo m" given by
 $R = \{ (x, y) / x - y \text{ is divisible by } m \}$ over the set of positive integers is an equivalence relation
3. Define LUB and GLB of subset of a poset. Give examples.
4. Define distributive lattice
5. Define the following
 - i) DFA
 - ii) NFA
 - iii) Transition diagram
6. Construct the truth table for $(p \wedge (p \rightarrow q)) \rightarrow q$
7. With the help of examples explain free and bound variables
8. Define incidence matrix . Give an example
9. Define a complete bipartite graph. Draw the complete bipartite graphs $K_{2,3}$, $K_{2,4}$ and $K_{2,5}$
10. Draw a
 - a) Hamiltonian graph which is not Eulerian
 - b) Euler graph which is not Hamiltonian
 - c) Graph which is both Hamiltonian and Eulerian
11. Explain rooted and binary trees with examples
12. State kuratowskis theorem.

(10 x 3 = 30 marks)

Part B

All questions carry equal marks.

13 a)

- i) State and prove De-Morgans laws on set theory

5marks

- ii) Using mathematical induction, show that (n^3+2n) is divisible by 3, for $n \geq 1$ 4marks

OR

(b)

- i) Let R be a relation on the set of all ordered pairs of natural numbers defined by $(x,y) R (u,v)$ iff $xv=yu$. Show that R is an equivalence relation 5marks

- ii) Consider the functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ Defined by $f(x) = x^2+3x+1$, $g(x) = 2x-3$, find the composition functions of
 i) $f \circ f$ ii) $f \circ g$ iii) $g \circ f$ iv) $g \circ g$ 4marks

14. a).

. Let $A = \{1,2,3,6,9,18\}$ Define R on A by xRy if x/y . Prove that A is a poset. Draw the Hasse diagram for the poset (A,R) 9marks

OR

b) Construct a deterministic finite automata accepting the set of all strings ending in 00 over the alphabet $\{0,1\}$ 9marks

15. a) Show that $(\exists x) M(x)$ follows logically from the premises $(x) (H(x) \rightarrow M(x))$ and $(\exists x) H(x)$ 9marks

OR

b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q \rightarrow R$, $P \rightarrow M$ and $\neg M$ 9marks

16. a) Show that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree 9marks

OR

b) Prove that a simple graph with n vertices and k components can have at most

$$\frac{(n-k)(n-k+1)}{2} \text{ edges} \quad \text{9marks}$$

17 a) State and prove Euler's formula 9marks

OR

b)

i) Prove that the minimum possible height of an n vertex binary tree is $\lceil \log_2(n+1) \rceil - 1$ 4marks

ii) Explain Prim's algorithm to find the shortest spanning tree of a graph

with an example

5marks

(5 x 9 = 45 marks)