# MSc. Mathematics Degree (MGU-CSS-PG) Examination

## (Model Question)

## Ist Semester

**PC4- MT01C01** **Graph Theory**

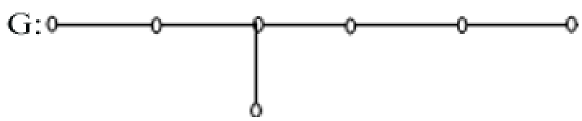
Time 3 hrs. Maximum Weight. 30

**Part A-Answer any five questions. Each question has 1 weight.**

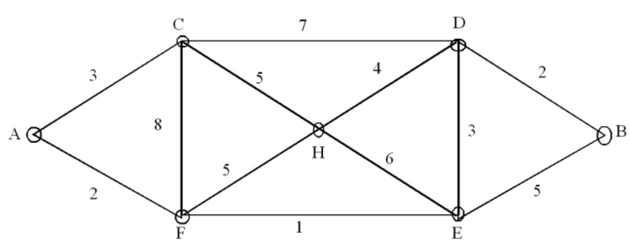
1. In any graph G, show that the number of vertices with odd degree is even.
2. Prove that an edge e= xy of a graph G is a cut edge of G if and only if, e does not belong to any cycle of G.
3. Show that every connected graph contains a spanning tree.
4. Prove that every tree is a bipartite graph.
5. Determine the values of the parameters $\propto, \propto^1, \beta, \beta^1$ for the Petersen graph P.
6. Write a note on Hamilton's "Around the World " game.
7. Show that a Hamiltonian cubic graph is 3-edge chromatic.
8. For any simple planar graph G show that $\delta(G) \leq 5$

**Part B-Answer any five questions. Each question has 2 weights.**

9. Define an identify graph. Show that the graph G of the following figure is an identity graph.




10. Show that every tournament contains a directed Hamiltonian Path.
11. Prove that the number of edges in a tree with n vertices is n-1. Conversely show that a connected graph with n vertices and n-1 edges is a tree.
12. Describe Dijkstra's algorithm for determining the shortest path between two specified vertices in a connected weighted graph. Using Dijkstra's algorithm find the shortest path from A to B in the weighted graph G of the following figure




13. State and prove Ore's theorem.
14. For every positive integer k show that there exists a triangle free graph with Chromatic number k
15. If G is a bipartite graph, then prove that $x'(G) = \Delta(G)$.
16. Define a planar graph and a plane graph show that a graph G is planar if and only if each of its blocks is planar.

**Part C-Answer any three questions. Each question has 5 weights.**

17. Define (i) bipartite graph and (ii) cycles in a graph. Show that a graph G with at least two vertices is bipartite if and only if it contains no odd cycles.
18. Show that a simple cubic connected graph G has a cut vertex if and only if, it has cut edge. Also show that the connectivity and edge connectivity of a simple cubic graph G are equal.
19. For a connected graph G prove that the following statements are equivalent .
    (i). G is Eulerian.
    (ii) The degree of each vertex of G is even.
    (iii) G is an edge disjoint union of cycles .
    Hence show that the Konigsberg bridge problem has no solution.
20. Show that $\tau$(Kn)=$n^{n-2}$ where Kn is the labeled complete graph.
21. For any simple graph show that $\Delta(G) \leq x'(G) \leq \Delta(G) + 1$
22. Define vertex colouring of a graph G and show that every planar graph is 5-vertex colourable.