MSc. Mathematics Degree (MGU-CSS-PG) Examination

MODEL QUESTION PAPER

SEMESTER I

PC 5, MT 01C05, COMPLEX ANALYSIS

Time: 3 hr **Max weight: 30**

Part A . Short Answer questions

Answer any five questions. Each question has 1 weight.

1 Suppose f(x) is analytic and satisfies the condition $\left|f^2(z)-1\right|<1$ in a region Ω. Show that either real part of f(z)>0 or real part of f(z)<0 throughout Ω

2If $T_1(Z)=\frac{Z+2}{Z+3}$ and $T_2(Z)=\frac{Z}{Z+1}$ find $T_1^{-1}T_2(Z)$

3 Compute $\int_\gamma xdx$ where γ is the directed line segment from 0 to 1+i

4If γ be a closed curve lies inside a circle then prove that $n(\gamma,a)=0$ for every point 'a 'outside the Circle

5If f(z) is analytic and non constant in a region Ω then prove that $\left|f(z)\right|$ has no maximum value in Ω

6Show that the number of zeros of f(z) inside the circle γ is $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}dz$

7How many zeros of $z^7-2z^5+6z^3-z+1$ lie in $|z|$<1

8Show by Rouche's theorem that the n roots of $e^z=az^n$ lie in $|z|$<1 where a>e

Part B . Short Essay questions

Answer any five questions. Each question has 2 weights.

9 Prove that cross ratio is invariant under a linear transformation

10Prove that conformal mappings are angle preserving

11 Prove that an arc γ,where length of arc z=z(t) , $a\le t\le b$ is rectifiable iff real and imaginary part of z(t) are of bounded variation

12 State and prove fundamental theorem of algebra

13State and prove Weierstrass theorem

14 If $f(z)$ is analytic at z_0 and $f'(z_0)$ is $\neq 0$, then prove that $f(z)$ maps a neighbourhood of z_0 conformally and topologically onto a region.

15 Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$

16 If 'u' is harmonic in Ω then prove that $f(z) = u_x - i u_y$ is analytic in Ω

Part C.Long Essay Questions

Answer any three questions. Each question has 5 weights.

17a) If $w = f(z)$ is angle preserving and $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are continuous then prove that $f(z)$ is analytic

b) Prove that the cross ratio of (z_1, z_2, z_3, z_4) is real iff. the four points lie on a circle or on a straight line.

18a) Find the linear transformation which carries the circles $|z| = 2$ into $|z+1| = 1$, the point -2 into the origin and the origin into i.

b) Prove that the line integral $\int_\gamma p dx + q dy$ defined in Ω depends only in the endpoints of γ iff. there exists a function $u(x, y)$ in Ω such that $\frac{\partial u}{\partial x} = p$ and $\frac{\partial u}{\partial y} = q$

19 a) State and prove Cauchy's Theorem for a disc.

b) State and prove Cauchy's Theorem for a rectangle

20 a) State and prove Taylor's Theorem.

b) If $f(z)$ is analytic in a connected domain Ω then prove that either $f(z)$ vanishes identically in Ω or $f(z)$ can never vanish together with its derivatives.

21 a)Prove that a region is simply connected iff. $n(\gamma, a) = 0$ for every cycle γ in Ω and for every 'a' not in Ω

b) State and prove Cauchy's Residue Theorem

22 a)Let $f(z)$ be meromorphic in Ω and a_j and b_j represent respectively the zeros and poles of $f(z)$ in Ω. Then show that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$ for every cycle $\gamma \sim 0 (\text{mod}\, \Omega)$ such that γ does not pass through a_j and b_k.

b) State and prove Poisson's Formula.

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